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A Survey on State-of-the-art Denoising Techniques for Brain Magnetic Resonance Images

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***Abstract*— The accuracy of the magnetic resonance (MR) image diagnosis depends on the quality of the image, which degrades mainly due to noise and artifacts. The noise is introduced because of erroneous imaging environment or distortion in the transmission system. Therefore, denoising methods play an important role in enhancing the image quality. However, a tradeoff between denoising and preserving the structural details is required. Most of the existing surveys are conducted on a specific MR image modality or on limited denoising schemes. In this context, a comprehensive review on different MR image denoising techniques is inevitable. This survey suggests a new direction in categorizing the MR image denoising techniques. The categorization of the different image models used in medical image processing serves as the basis of our classification. This study includes recent improvements on deep learning-based denoising methods alongwith important traditional MR image denoising methods. The major challenges and their scope of improvement are also discussed. Further, many more evaluation indices are considered for a fair comparison. An elaborate discussion on selecting appropriate method and evaluation metric as per the kind of data is presented. This study may encourage researchers for further work in this domain.**

***Index Terms*— Magnetic resonance imaging, biomedical image denoising, brain MRI.**

I. Introduction

MAGNETIC resonance (MR) imaging is a trusted modality in clinical image diagnosis. The flexibility in the imaging modality provides better structural features of an organ. MR imaging facilitates multi-modal projection views with sectional images of equivalent resolution. The fundamental steps in MR image processing includes enhancement, registration, segmentation, object recognition and so on. Many methods have been reported for each of the steps. One of the essential step in enhancement is denoising. In general, noise in the MR image is characterized in terms of Rician distribution. However, MR images with low and high SNR values are characterized by Rayleigh pdf and Gaussian pdf, respectively. Further, noise in complex MR images is expressed in terms of additive white Gaussian noise. Noise in MR image increases the complexity of the diagnosis process for distinguishing the features. The noise is introduced due to erroneous imaging environment or processing in noisy transmission systems [1]. The effect of noise is observed as blurring regions, random variations, unrealistic edges and artifacts. Further, presence of indistinguishable anatomical boundaries possessing significant information and low spatial resolution degrades the performance of computer analysis [2]. Therefore, eliminating noise and preserving the edges without introducing artifacts are the basic requirements in any denoising procedure. There are many popular image denoising methods which can also be used for MR image denoising. For instance, mean, median, wiener, diffusion, domain and range based filters, stochastic and graph based approaches in spatial domain as well as transform domain. In a broad sense, we can classify the MR image denoising methods into two approaches: A. Hardware approach, B. Software approach. In the first approach, the noise elimination is achieved by improving the performance of MR scanning device. The patient may be scanned iteratively and then the mean value is taken. However, the performance of this approach is limited due to its large time averaging over repetitive attainments and limitation in accessing the hardware of the device. Further, the acquisition time makes the patient's comfort and imaging dynamic applications impracticable.

In the second approach, the images are denoised with a suitable software based post-acquisition scheme on the recorded data. This is an effective alternative for denoising and improving the visual clarity of an MR image. In this survey, we focus on the software approaches. The software approaches are further classified based on the medical image modelling categorization. The main philosophy of the proposed categorization is to simplify the complex system into a systematic representation. Further, it may be helpful in formulating hypotheses, organizing critical experiments, and a precise way for investigating general or specific quantitative phenomena. To have a systematic representation, a tree structure of various methods used in MR image denoising is presented in Fig. 1. The methods are categorized into spatial domain and transform domain. In transform domain, the approaches are grouped into data adaptive and non-data adaptive techniques based on the choice of basis functions. The data-adaptive techniques use independent component analysis (ICA) for noise elimination. The non-data adaptive techniques are further grouped into frequency domain and time-scale (wavelet) domain.

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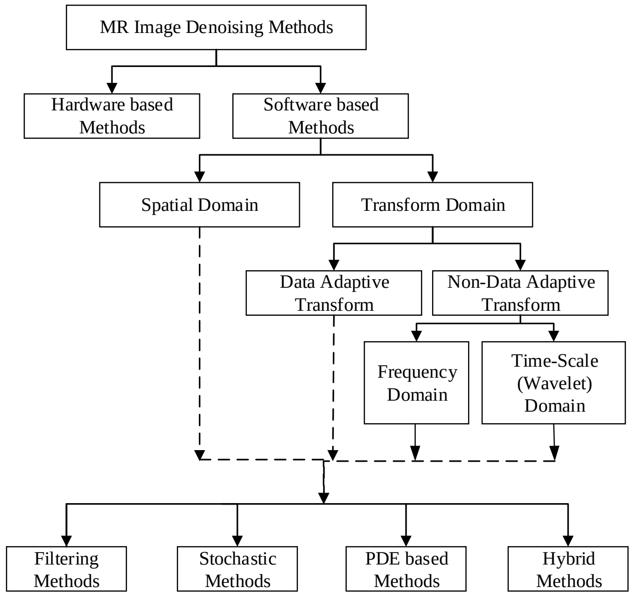

Fig. 1. Classification of MR image denoising methods.

A further classification into filtering methods, stochastic methods, partial differential equation (PDE) based methods and hybrid methods can be implemented in both spatial or transform domain. Although, the presentation is classifying the different methods, it is to be noted that they are not exclusively different. Further, some methods may have interrelation.

It is observed from the study that most of the research work is confined to a very limited sphere of methods used for denoising. The focus of most of the surveys is either on a specific MR image modality or on limited techniques used in reducing the noise. Further, a few evaluation indices are used for comparing the different denoising methods. That's why we are motivated to carry out yet another survey on the denoising schemes. The categorization of image models used in medical image processing serves as the basis of our classification scheme. This categorization may suggest a helpful way in further research formulation and conducting critical experiments. An elaborate discussion on the challenges and the future scope of different denoising schemes is incorporated in this study. The merits and demerits of the different denoising schemes are summarized in a tabular form. Many standard evaluation indices are included for quantifying the performance of different denoising techniques. Further, this survey may help readers in selecting appropriate method and evaluation metric as per the kind of data, and effect of noise levels in different denoising approaches. An elaborate detail on this aspect is presented in the discussion section. It is believed that, there is a lot of scope for improving the denoising performance. This may help researchers in solving the inherent problems of different schemes used in MR image denoising. The rest of this manuscript is organized as follows: The existing techniques on MR image denoising are discussed in Section II. In Section III, the validation measures used to evaluate the different denoising methods are discussed. Section IV presents a comprehensive discussion on comparison of the denoising schemes. Finally, the survey is concluded in Section V.

II. MR Image Denoising Techniques

In recent years, many researchers have presented survey on MR image denoising techniques [4]-[11]. Mohan *et al*. [4] in their study, categorized the denoising methods into three groups based on filtering approach, transform approach and statistical approach. The authors discussed comprehensively on the filtering approach. However, little discussion is done on the other two approaches. Further, recent techniques on deep learning based schemes are also not included in the study. Additionally, very few evaluation metrics are used while comparing the different denoising techniques. Bhujle and Vadavadagi [9] presented a survey on the denoising techniques using the nonlocal mean (NLM) filtering approach only. They did not consider any other approach in their study. Garg and Juneja [10] presented a survey on the denoising approaches for multi-parametric prostate MR images i.e. diffusion weighted and T2 weighted MR images with Gaussian noise. Their study concentrated only on the filtering approach of denoising. Further, their survey is more of application specific. Goyal *et al*. [11] presented a survey on the denoising techniques based on the noise models. The authors considered the Gaussian and the Rician noise model only, which is a very common noise in MR images. Further, they discussed primarily on the spatial filtering and wavelet domain filtering approaches of denoising with a single performance metric for comparison. The authors did not consider any other approach or metric for a fair comparison. The following sub-sections describe the various denoising schemes as shown in Fig. 1.

A. Data Adaptive Transform

The data-adaptive transform based approaches use ICA algorithm for noise elimination. The algorithm is used for revealing statistical independent factors and blind source separation. ICA is a computation method for denoising the multidimensional MR data. This is a self-adaptive higher order statistical tool for modeling a computer vision system. McKeown *et al*. [12] suggested the ICA based technique for denoising functional MR images. The method is an effective tool on denoising random noise, eliminating pulsation and breathing artifacts. Sukhatme and Shukla [13] suggested ICA as a pre-processing approach with Eigen value decomposition and dimensionality reduction. It maximizes the mutual information, while minimizing the non-Gaussian noise in MR images. Pignat *et al*. [14] suggested the ICA technique in wavelet transformed image for improvising denoising performance. The method decomposes the spatial image into its corresponding wavelet coefficients. Then, ICA is employed for eliminating Gaussian noise. The method is also effective in enhancing edges in the image. However, denoising performance and computational complexity are the limitation in these approaches. They can be improved by incorporating appropriate optimization algorithms.

B. Non-data Adaptive Transform

These approaches are formulated using the frequency transforms, wavelet transforms (WT). In this domain, the noise elimination and structural preservation are achieved

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simultaneously.

1) *Frequency Domain*

Frequency domain representation of MR images is attained using Fourier transform (FT). It reduces the small structure at the edges. A trade-off between the spatial information preservation and noise reduction is reported in [15]. In [16], the authors suggested FT based filtering for denoising MR images. The method used power spectra for estimating noise and measuring the standard deviation. Luo *et al*. [17] suggested a singularity function based reconstruction approach for denoising MR images. In their method, the image is first divided into a number of spectral units. Then, the denoising mechanism is applied on each spectral unit using 2D singularity function analysis. The denoised image is obtained through averaging reconstruction. Mustafi and Ghorai [18] suggested fractional FT based technique for denoising the medical images. The technique shows suitable characteristic for denoising the images with highly sensitive edges. It is also useful for blind source separation. In future, the denoising performance and edge sensitivity can be enhanced using the multistate nature of fractional FT.

2) *Time-Scale* (*Wavelet*) *Domain*

A wide variety of time-scale (wavelet) based MR image denoising schemes are reported in several articles in scientific and engineering journals. This transform decomposes the MR image into sub-bands of wavelet coefficients ranging from the roughest to the fine details. The coefficients with small absolute magnitude are usually noise or small structures at the edges in an image. Removing such values reduce the noise as well as the fine details in the reconstructed image. However, the selection of an accurate threshold value preserves the fine details, while improving the denoising performance. Xu *et al*. [19] suggested a spatially correlated noise filtration technique in the WT domain for MR image denoising. The regions with higher spatial correlation are associated with several adjacent scaling coefficients. Nowak [20] suggested a WT based denoising scheme for Rician distributed noise in magnitude MR images. Zaroubi and Goelman [21] suggested a complex denoising scheme for denoising MR images. It is achieved by shrinking noisy discrete wavelet coefficients using soft thresholding. The noise elimination is carried out by decomposing the image into two sets of orthogonal wavelet coefficients. Bao and Zhang [22] suggested the multiscale thresholding of wavelet coefficients with Canny edge detector for noise elimination. Wink and Roerdink [23] suggested WT based denoising technique for functional MR images. The method used 1D WaveLab thresholding in 2D wavelet coefficients.

Wu *et al*. [24] suggested WT based technique for removing the Rayleigh distributed background in MR images. The wavelet coefficients are represented as non-stationary data. The uncorrelated noisy background is separated by scaling the wavelet coefficients. In [25], the authors suggested AWT approach for denoising MR images. The methods are adaptive to noisy data and SNR variations in the MR images. The correlation among the resolution scale is used for estimating the noisy wavelet coefficients. In [26,27], the authors suggested the bilateral filter (BF) in the WT domain for denoising MR images. This filtering approach effectively eliminates Rician noise while preserving edge features. Bartusek *et al*. [28] suggested an optimized WT (OWT) based technique for MR image denoising. The approach is intended for optimizing the threshold levels and selecting the mother wavelet. Luisier *et al*. [29] suggested the undecimated filter bank in wavelet domain for estimating the noise in MR image as non-centrality differentiable chi-square random variable.

Habiba and Raghu [30], suggested dual tree complex threshold function in WT for denoising random noise in MR images. The method is employed for denoising infinite dimensional objects, such as: lines, curves etc. Dual tree complex thresholding function and WT are combined for successful balancing of smoothness and accuracy. Agarwal *et al*. [31] provided a comprehensive comparison of different WT schemes with random noise in MR images. In [32], Naveed *et al*. suggested a goodness of fit test on the WT coefficients of the noisy MR image. It employs an Anderson Darling statistics in the goodness of fit test context for computing the noisy WT coefficients. However, use of local noise variance for optimizing the denoising performance can be taken as a future direction in this approach. Further, WT can be replaced with some other transform which is rotational, translational and shift invariant.

The wavelet transforms are not suitable for analyzing images with high dimensional edge structures. Wiek and Figiel [33] suggested curvelet transform (CVT) for denoising the brain MR images with high dimensional information content. The edge information in this transform is represented using theory of multiscale geometry. The frame features are represented using the position and scaling of the edges. The sparse representation of confined CVT frames facilitates Fourier integration and virtual differentiation operator. Bhadauria and Dewal suggested [34] an approach by combining the CVT and total variation method for denoising brain MR images. The structural details in the MR image is extracted from the residual noise component using the CVT technique. Vanitha *et al*. [35] suggested CVT for reducing the fractional Brownian motion noise in medical images. The thresholding schemes, such as: BayesShrink, NeighShrink and VisuShrink in combination with the curvelet transforms were experimented for effectively denoise MR images. Biswas et al. [36] suggested a wiener filter based CVT technique. This transform decomposes the image into disjoint scaling using a local Ridgelet transform. The CVT based techniques with a suitable thresholding are found effective for eliminating Rician noise in MR images. However, finding optimal threshold values can be a future scope for improving the denoising performance of this approach.

Contourlet transform (CNT) is an extension of CVT that represents the multi-dimensional multi-resolution features in an image. This transform uses Laplacian pyramid and directional filter bank for decomposing the contourlet in specific frequency bands. The directional decomposition facilitates the allocation of different orientations and scaling at various resolution in the image. Satheesh and Prasad [37] suggested different thresholding approaches with CNT for denoising MR images. The method is implemented with different soft and hard thresholding approaches removing the Gaussian noise. It also provides an effective representation of high directional

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anisotropic textural features. Kazmi *et al*. [38] suggested thresholding based CNT techniques for brain MR image denoising. The transformation is accomplished by two successive decompositions, as: multi-scale and multi-directional. A Laplacian pyramid is used for the multi-scale decomposition for generating a set of low-pass and band-pass images. Further, directional filter bank is used for multi-directional decomposition of each band-pass image into critical sub-samples. It is to be noted that finding optimal threshold values can be a future scope for this denoising approach also.

In sparse representation, the actual values of data points are reconstructed from the linear combination of the basis functions i.e. the sparse representation of the data points. Grouping of similar data blocks into a stacked array is called block-matching 3D (BM3D). It provides a collaborative nonlinear filtering approach for eliminating the noise from the complex MR images, while preserving the edge information. Lin and Qiu [39] suggested the sparse representation for noise removal in transform domain. In the process, similar 2D blocks are grouped to form the 3D data array for sparsity enhancement. Further, collaborative filtering is used for preserving the unique features of each block, while eliminating the spurious noise. In [40], the authors suggested a modified BM3D (BM4D) approach for eliminating noise in MR images. The conventional BM3D technique is employed with the wavelet shrinkage for improvising the denoising capability. However, these approaches can be improved by combining with the CVT and CNT methods for denoising multi-dimensional MR images. It may be concluded from the above discussion that the transform domain approaches are found to be effective for denoising Rician noise in MR images as compared to spatial domain methods.

C. Filtering Methods

Here, a weighted kernel is used for modifying the pixel intensities. The convolution of the noisy image with the weighted kernel reduces the noise by decreasing the variance in the image. The used kernel may be linear or non-linear. Accordingly, the filtering methods are further classified into linear and non-linear filtering as shown in Fig. 2. Linear filtering is implemented using smoothing and temporal filters [41]-[46] for removing uniformly distributed noise. On the other hand, nonlinear filters are used for denoising images with unevenly distributed noise. Among the nonlinear filtering approaches, the anisotropic diffusion (AD) [47]-[53], NLM [54]-[63], BF [64]-[69] are considered in this study.

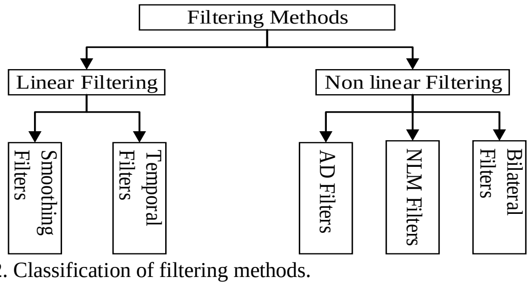

Fig. 2. Classification of filtering methods.

1) *Linear Filtering*

The smoothing filter approach uses a smoothing function for denoising the Gaussian noise (uniformly distributed noise) in an image. The convolution of the noisy image with a smoothing kernel decreases the Gaussian noise effect by reducing the variance in the image. However, fine details in the image are blurred due to the weight factor associated with the parameters used in the filtering kernel. Such techniques are implemented for reducing the elevated spatial frequencies in an image. In general, the smoothing filtering is achieved using the mean, median, wiener filters and their modifications. McVeigh *et al*. [16] suggested the wiener filters (WF) for reducing the Gaussian noise in MR images. They assumed that the noisy image contains higher values of spatial frequencies. The denoising techniques using such filters improve the SNR value. However, the feature details at the image edges are eliminated due to the blurring effect resulting in reduced clarity of the MR image.

Coupe *et al*. [41] suggested adaptive median absolute deviation estimator for denoising Rician noise. Mohan *et al*. [42] proposed an extension of the median filter (MF) utilizing a directional window. The spatial structural characteristic is used for edge preservation from the max and min median values. Bin-Habtoor *et al*. [43] proposed a cascade of mean and adaptive median filtering (AMF) for denoising speckle noise. However, the authors focused only on the edge regions using their cascaded approach. Kadam and Borse [44] proposed a spatial adaptive filtering for denoising the MR images. However, the denoising performance of the technique is limited to salt and pepper noise only. Ali [45] suggested the adaptive median and WF for eliminating the additive Gaussian noise in MR images. Seetha and Raja [46] compared different filtering based methods used in denoising MR images. The quantitative analysis shows the superiority of the AMF to adaptive wiener filter (AWF) in eliminating the additive Gaussian noise. The performance of all the smoothing filters is limited due to the elimination of small features. However, the denoising performance can be improved using an edge-preserving tool.

Temporal filters are designed for eliminating temporal variations in an image. The temporal variations in the image sequences, such as rapid variation, spin echo effects and object movement are reduced by filtering the sampling intervals. Moreover, the sampling intervals are intuitively chosen for eliminating the aliasing noise. McVeigh *et al*. [16] suggested temporal filtering the MR images with different sampling intervals. The filter eliminates the narrow frequency components in the image. In the process, the signal components are also lost in the same narrow band, i.e. from the edges in the image. This also introduces aliasing noise at the broader frequencies resulting in a smoothed image. However, the SNR remains unaffected, because the noise as well as the signal reduces by the same factor. However, the denoising performance of this filter can be improved by combining it with the parametric stochastic methods.

2) *Nonlinear Filtering*

AD filters use diffusion in homogeneous regions while preventing diffusion at the edges. It does not require the image details prior to the denoising process. These criteria make such filters popular among the non-linear filters. These filters overcome the difficulties of smoothing filters used in denoising along with preserving the object boundaries. AD filters result in selective diffusion on the local information from the neighboring pixels. They compute the group diffusion as a single diffusion value of its neighborhood by summarizing diffusions in each time step. The filter is based on 2^{nd} order PDE (2PDE). Gerig *et al*. [47] proposed a nonlinear AD filtering method for denoising the 2D dual echo spin and 3D gradient echo MR images. The method is implemented for minimizing the information loss by preserving object details. However, the model is piecewise persistent and gradually varying resulting in sharp edges with a steady intensity slope.

Murase *et al*. [48] proposed diffusion filtering of dynamic susceptible MR images for eliminating the Rician noise. The method is experimented in highly noisy environment of dynamic susceptible MR image. Samsonov and Johnson [49] proposed a noise adaptive AD filter scheme for eliminating Rician noise in MR images. This approach extracts the information content present at the edges in the T2-w MR image. It also smooths the intra region of the MR image depending on the differential structure. The resulting image is a multi-scale smoothed image with preserved fine details. Further, the authors proposed a non-linear AD filtering approach for spatially changing noise levels in MR images. They introduced a spatial noise distribution factor for eliminating the spatially changing noise levels [50]. Krissian and Aja-Fernandez [51] suggested a noise-driven AD (NDAD) filter for denoising Rician noise from MR images. The filter is dependent on the estimation of the standard deviation of noise and PDE of the MR image. It leads to the development of a coherent diffusion matrix based on the structural property of the image. The resulting filter is experimented for improving the rate of convergence while preserving the information content at the edges. Pal *et al*. [52] suggested a moment based AD filter for eliminating the Rician noise in MR images. The AD filter is remodeled by incorporating a diffusion coefficient. This is computed from the 2^{nd} order moment of the Rician noise. Cappabianco *et al*. [53] suggested an operational AD filter for MR image denoising. It is a modification of the standard AD filtering model by incorporating a diffusive conductance factor. The edge stopping function used in the process preserves the structural details in the edges of the image, while diffusive conductance factor makes it suitable for noise estimation and contrast enhancement. However, this filtering scheme can be developed in future for processing the multichannel diffusion tensor MR images.

It is observed from the literature that the use of local filtering results in large-scale structural preservation but elimination of finer details. To overcome this problem, the NLM filtering is experimented to exploit the redundant information in an image. This is accomplished by incorporating the mean of all pixels of an image with similar weight into the target pixel. Manjon *et al*. suggested different NLM based filtering schemes for eliminating Rician noise in MR image [54]-[56]. In [54], the authors suggested an unbiased NLM filtering approach for finding the optimal parameter in denoising magnitude MR images, while keeping the structure distinguishable. Instead of the pixel similarity comparison, they introduced a region based similarity comparison. This process makes it independent of the local pixels and more robust for denoising. Further, the authors also suggested an adaptive NLM filtering based approach for denoising the MR images with spatially fluctuating noise levels [55]. Then, the authors suggested denoising techniques using the sparseness and self-similarity behavior in the MR images. The techniques are formulated on 3D moving window based on cosine transform and 3D rotational invariant form of NLM filters [56]. However, the performance of such approaches is limited due to their computational complexity. Coupe *et al*. [57] suggested a fast NLM (FNLM) scheme for reducing the computation time in denoising MR images. The authors extended their work towards automatic and optimized blockwise NLM (OBNLM) filtering scheme for denoising 3D MR images [58]. It is achieved by a block wise implementation in parallel computing mode. In comparison to the conventional approach, this filter reduces the execution time significantly. Further, the OBNLM filtering with wavelet-based thresholding is used for denoising multiresolution MR images [59]. However, the performance is limited due to redundant spatial information. Liu *et al*. [60] suggested a Gaussian based undecimated NLM (UNLM) filtering scheme for eliminating biased deviations in 3D MR images. The denoising is carried out using the weighted average of gray levels in the global area. Hu *et al*. [61] suggested a denoising mechanism using NLM filter based on the discrete cosine transform (DCT). The method computes similarity measures in DCT space. This is modeled for eliminating the distortions due to noise and improving the computational complexity. However, similar textures in brain MR image may have different information content.

Chen *et al*. [62] suggested a repetitive structure of collaborative NLM filters for denoising MR images. The structures from multiple scans are aligned to provide better features than the single scan. Further, the block matching in NLM enhances the edge preservation. Yu *et al*. [63] suggested a Laplacian Eigen map network based NLM filtering for reducing the noise in MR images. It is achieved by computing the similarity in extracted features. However, in future, the NLM filtering technique can be improved using textural features and optimization algorithms for obtaining better denoising performance with mix noise.

BF is a non-linear approach of combining the nearby gray levels based on their geometric and photometric similarity. This filtering approach is introduced by Walker *et al*. [64] for smoothing images with nonlinear combination of neighboring pixels. The method uses the photometric similarity property for combining the gray levels or color scale on their geometric nearness, preferably nearer to distant values in domain and range. It incorporates the perceptual metric by smoothing gray levels and preserving edges. Mustafa and Kadah [65] suggested multiresolution BF (MRBF) for approximating sub-band decomposition and reconstruction in wavelet domain. The sub-band decomposition using wavelet thresholding flattens the

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gray levels and eliminates high frequency noise. Lin and Chang [66] suggested a parametric optimization technique using backward propagated artificial neural network (ANN) for optimal performance of BF approach. The ANN based parameter optimization uses the statistical and gray level co-occurrence matrix features. Phophalia and Mitra [67] suggested rough set based BF (RSBF) for denoising Gaussian noise from MR image. The filter introduces a boost-up parameter of spatial nearness. The parameter is adaptive in terms of rough class label and edge map of the noisy MR image.

A modification on BF for identifying the non-homogeneous regions in the image is a trilateral filter (TF). The method integrates local structural similarity in addition to geometric and photometric similarities for smoothing images. The method uses a narrow spatial window and takes only one iteration for smoothing the images. It is also experimented on multidimensional signals. In this method, the pixel values are replaced with the average values of the weights from geometric, photometric and local structural similarity in the neighborhood [68]. Chang *et al*. [69] proposed a modified TF for removing the fluctuation due to Rician noise. The filter uses the rank ordered absolute difference statistics based on extreme compression for eliminating the Rician noise in MR images. However, the filter parameter selection and optimization are the scope of further improvement in this approach. Further, the method can be improved in the direction of adaptive 2D filtering for random noise types.

D. *Stochastic methods*

The methods are inherently random in nature. This makes the stochastic methods more rational for processing the MR images in the noisy environment. A set of initial conditions and parameter estimation is prepared for modelling various estimators. The methods are used to estimate the noise characteristics and their variance property prior to the denoising process. Using a nonlinear function, the data can be modelled as the sum of the clean data plus additive Gaussian or Rayleigh noise. The Laplacian probability density function can be used for the clean data in the transformed domain. A prior distribution accurately characterizes the heavy-tail distribution of clean images and exploits the interscale properties of the transform coefficients. In addition, the parameters of the model can be estimated by using local information, thus, making the denoising algorithms spatially adaptive. A suitable model is chosen as per the noise variance characteristics in the MR images. Based on their population estimation criteria, the stochastic methods are categorized as shown in Fig. 3.

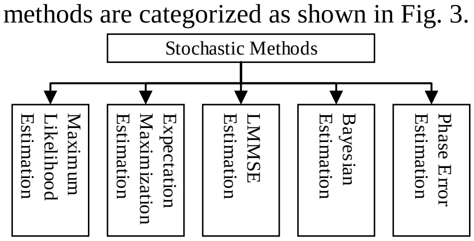

Fig. 3. Classification of stochastic methods.

1) *Maximum Likelihood Estimation*

Maximum likelihood (ML) estimation is a method of computing the parameters from the probability distribution of a finite Gaussian model. This is the value that maximizes the likelihood function in the parameter space. Sijbers *et al*. [70] suggested different approaches using ML estimation for Rician noise removal and image reconstruction. The ML estimation is formulated using two-stage acquisition method for obtaining noise variations. The method computes the amount of noise by subtracting two successive acquisitions of the same object. It reduces the bias effects that appear in the conventional ML estimation. Further, in [71,72], the authors suggested parameter estimation for complex valued MR data using ML method. Here, the Gaussian noise distribution in the complex MR image is remodeled in Rician distribution using its magnitude components. He and Greenshields [73] suggested nonlocal ML (NLML) estimation for Rician noise in magnitude MR image. It has taken the assumption that the pixels having similar neighborhood are from the same distribution. This method uses ML estimator in the nonlocal neighboring pixels for predicting the underlying noise. In [74-77], Rajan *et al*. suggested different models for estimating the noise level in an MR image without background. The noise variance is computed using the ML estimation and the local skewness. The method gives an effective solution for denoising magnitude MR image in the absence of the background. In [74], the authors suggested local ML (LML) estimation for denoising Rician distributed magnitude MR images with restricted local neighboring pixels. In [75], the authors suggested NLML estimation for estimating true information from MR images obtained from phased array coils. The method estimates the true information from the root sum square of the non-centrally distributed data. In [76,77], the authors suggested a similarity based NLML estimation for denoising MR images. The gray level similarity is computed using the Kolmogorov-Smirnov approach.

2) *Expectation Maximization Estimation*

The parameter estimation using expectation maximization (EM) method estimates the ML of a Gaussian mixture model in presence of latent variables. This method computes the latent variables and optimizes the model iteratively. Therefore, the computation of noise variance is avoided in the parameter estimation process. Maitra and Faden [78] suggested the EM technique for parameter initialization and variance estimation in denoising magnitude MR images. The parameter estimation is independent of the number of background pixels in the noisy image. Martin-Fernandez and Villullas [79] suggested a probabilistic wavelet transform based denoising scheme for brain MR images. The model helps in merging two Gaussian distributions without deviating from the actual distribution. Further, the computational complexity is reduced by the iterative learning process.

3) *Linear Minimum Mean Square Error Estimation*

Brain MR image denoising using LMMSE is a versatile approach of minimizing the mean square error using quadratic loss function. The approach is found to be effective in the regions of image having dependent pixels. Aja-Fernandez *et al*. [80,81] suggested the use of LMMSE estimation for minimizing Rician noise in the diffusion weighted MR images. These methods use the local statistical sample distribution for computing the noise power in an image. The actual value of each pixel is estimated using the adaptive LMMSE (ALMMSE) estimation from its local neighboring spatial information. Further, the dynamic noise estimation using the local statistical sample distribution is found effective for denoising and feature preservation in noisy MR images.

Golshan *et al*. [82] suggested a signal dependent biased LMMSE estimation method for eliminating Rician noise in magnitude MR image. The method is a nonlocal LMMSE estimation approach for computing controlling parameters with the use of hard thresholding. Further, they suggested a modified recursive LMMSE (RLMMSE) estimation for denoising 3D MR images [83]. The method incorporates nonlocal neighboring spatial information for estimating the samples from the MR image assuming a random field. The structural similarity measure of the model is improved by considering the degree of redundancy in the 3D MR image.

4) *Bayesian Estimation*

This is a stochastic approach of estimating the true value of a pixel from its neighborhood in the absence of supporting parameters. Therefore, the method is also called as nonparametric estimation technique. Awate and Whitaker [84] suggested a nonparametric estimation method for denoising MR images. Considering the images as random fields, the method computes the higher order coherent data from the noisy MR image using reduction coupled Rician noise model. The neighboring structure is characterized using nonparametric density estimation for Bayesian denoising of MR images. Further, they [85] suggested an extended nonparametric empirical Bayesian method for preserving features and denoising MR images. Their method computes the denoised image statistics from the MR image with Rician distributed noise. This prior is estimated by an optimized data metric using EM technique. The generality of the empirical Bayesian approach for the prior estimation provides fitting methods for denoising and feature preservation in MR images.

Rubio and Nunez [86] suggested a general approach of kernel regression framework for denoising Rician noise in MR images. The method is a zeroth order kernel regression for computing a weighted average in a regression window. The weighted average values are computed from the similar feature vectors associated with each data point. These features are characterized as second order kernel regression of true data point and its gradient vectors. This directional information substantially enhances the filter performance for denoising and feature preservation. Wong and Mishra [87] suggested a stochastic approach using quasi-Monte Carlo technique for estimating noise free image. The statistical characteristic of the noise free image is expressed as Bayesian least square estimation problem. The regional statistics for noise estimation is obtained in a data adaptive method. Monir and Siyal [88] suggested an anisotropic spatial averaging based approach for denoising functional MR images. The smoothing function is computed for every pixel in time-series. The method is independent of the test hypothesis information, while it is found to be suitable as a pre-processing stage for both hypothesis and data driven analysis. These methods use different adaptive median absolute deviation estimation in the detailed components in the wavelet-transformed image for eliminating the Rician noise.

5) *Phase Error Estimation*

Phase error estimation is a model based iterative restoration procedure. It computes the maximum a posteriori (MAP) estimation of phase and reflectance of speckle free object. The phase estimation of the noisy image is computed using a series of non-linear filters. The phase correction of each data point is reconstructed using the estimated phase error. The imaginary component in the estimated phase error is the noise contaminant in a noisy image. This can be eliminated easily. This procedure indicates the effective preservation of edges in the MR images. Tisdall and Atkins [89] suggested a phase error estimation based method for denoising MR images with low SNR values. This shows better edge preservation in comparison to the other nonlinear filtering approaches, such as: AD, NLM filtering. This is a potential denoising method for complex valued MR images without the risk of over smoothing.

E. PDE Based Methods

The higher-order PDE (HPDE) filters are suggested for minimizing the absolute value of image intensity's Laplacian function. The resulting images appear natural than the step images obtained using the 2PDE. Lysaker *et al*. [90] suggested smoothing based 4^{th} order PDE filter for eliminating the artifacts due to the staircase effects in MR images. Their technique is experimented in both space and time domain. It shows better results for the blocky effects in the smoothly changing grey values. Jin *et al*. [91] suggested a modified HPDE filtering for denoising MR images using the consistency of a pixel with its neighborhood. The pixel similarity measures the belongingness of a pixel in its neighborhood, thereby reducing the noise. Rajan *et al*. [92] suggested nonlinear complex diffusion approach in HPDE filtering for noise elimination in MR images. The nonlinear complex diffusion approach makes it effective in reducing noise as well as preserving information details in edges. Khanian *et al*. [93] suggested an optimal PDE filtering technique for MR image denoising. The method is based on a new stopping criterion using the higher frequency relative difference factor in a region. Jansi and Subashini [94] suggested a PDE based on Rudin-Osher-Fatemi filter for denoising Rician noise in MR images. The method also eliminates the speckles from the edge regions by discontinuity treatment. Heydari and Karami [95] suggested modified diffusive function using pixel similarity in the neighborhood. Kollem *et al*. [96] suggested an adaptive 4^{th} order PDE filters for denoising Rician noise in MR images. The filter is designed to reduce the execution time of the PDE by using the gradient and Laplacian function. Considering the whole image to be planar, the PDE approach attempts to reduce the noise while preserving the edge regions. However, these techniques can be further improved in smoothing out the higher frequency components while retaining the structural details in the highly noisy images.

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F. Hybrid Methods

Besides the discussions presented above, few studies are reported in the literature that uses a hybrid approach for denoising. The hybrid approach suggested by Liu *et al*. [97] is a combination of fuzzy clustering and NLM filtering for denoising brain MR images. The method is a patch-based approach for denoising structural redundancy in MR image. Ma and Plonka [98] suggested the hybridization of nonlinear AD filtering with CVT for preserving the edge discontinuity while denoising. Aravindan and Seshasayanan [99] suggested discrete WT in combination with monarch butterfly optimization algorithm for MR image denoising. Coupe *et al*. [100] suggested an automated NLM filtering approach in combination with the wavelet sub-band mixing for image restoration while preserving relevant image information. The filtering is improved using the multi-resolution approach. This is to make the smoothing parameters adaptive along frequency variations. Rabbani *et al*. [101] suggested a Laplacian mixture model in WT domain for reducing the Rayleigh distributed noise in MR images. The used statistical model helps in reducing the distortion in the denoising process. Ashamol *et al*. [102] suggested a hybrid approach combining the stationary WT (SWT) and CVT with AD filtering for denoising Gaussian noise in MR images. The SWT is good at representing point singularity and small patches, while CVT are sparse representation based multiscale transforms. Thus, the combined effort of these two transforms preserves better information in comparison to the individual transform. Further, AD filtering is reducing the pseudo-Gibbs artifacts. Kala and Deepa [103] suggested a hybrid algorithm, combining a spatial domain BF and optimized thresholding in WT domain for denoising MR images. The first stage spatial BF is used for denoising the low frequency sub-band of the decomposed image. The second stage BF denoise the high frequency components present in the noisy image.

Zeng *et al*. [104] suggested a sparsely denoted hybrid approach for denoising MR images. The first one is the morphological WT coefficients and second is the sparsely represented texture based residual component. However, the denoising performance is not worthy enough. Besides these, there are optimization algorithms widely used in combination with the denoising schemes. These hybrid algorithms are effective in comparison to the denoising techniques used alone. Further, denoising models with optimal parameter increases the linearity with noise variance in an image [105]. Manjon *et al*. [106] suggested a nonlocal principle component analysis (PCA) based thresholding approach for estimating the local noise in combination with the rotational invariant form of NLM filtering for denoising images with spatial changing noise levels. This technique is experimented for rectifying the effects of non-stationary Rician noise introduced locally in an MR image.

Chang *et al*. [107] suggested parameter optimization of the NLM filtering technique using PCA for 3D MR images. The method rearranges the data points in a decreasing order of the variance in the image. This disassociates the signal components from the noisy MR image by eliminating the noise components. Sudeep *et al*. [108] suggested the LMMSE based NLM filtering for denoising MR images. The similarity weights in NLM are estimated from the Euclidean distance between the pixels in the spatial domain. The similarity weights in NLM are estimated from the Euclidean distance between the pixels in the spatial domain. Assuming the signal dependent components are correlated, the uncorrelated noise elements are suppressed in the image. In [109], the author suggested BF technique for eliminating Rician noise in an MR image, while preserving edges. Usually, the performance of the approach is limited due to non-optimal parameters. This is achieved using Genetic Algorithm (GA). Jiang *et al*. [110] suggested a feed forward neural network based learning approach for denoising non-stationary Rician noise in brain MR images. This is employed for reducing the computational complexity in parameter estimation using optimization algorithms. Ran *et al*. [111], suggested a Wasserstein generative adversarial network based approach for denoising MR images. Benou *et al*. [112] suggested a spatio-temporal approach for denoising dynamic contrast enhanced MR images. The method uses a deep neural network for establishing a deep auto encoder, where the training dataset is formed using different noise features. Xie *et al*. [113] suggested a deep learning based approach for denoising arterial spin labeling MR images. The method uses convolution neural network in combination with wide activation residual blocks for improving the denoising performance. Li *et al*. [114] suggested a distribution based neural network for eliminating Rician noise in MR image. The method consists of two residual blocks, one is for pixel domain fitting and the other is for feature domain matching. These blocks are used for formulating a progressive network learning procedure. However, these approaches increase the computational complexity and execution time in restoring the denoised image.

Liu *et al*. [115,116] suggested a total variation based feature preserving denoising scheme for eliminating Rician noise in MR images. The method is a two-step WT domain approach for extracting spatially varying Rician noise map in the MR image. Further, the authors suggested a local variance estimation for computing the spatial adaptive regulating parameters. However, the computational cost of the non-local regulating parameters is a potential constraint in these methods. Pieciak et al. [117,118] suggested variance stabilized transformation approaches for denoising non-stationary Rician noise in MR images. These methods are analytically derived from the unbiased NLM filtering to a closed form of return to origin probability map. However, the methods are dependent on the initial values of the noise variance and SNR. Table I summarizes the merits and demerits of the various MR image denoising approaches along with the period of introduction and the relevant references. The method description could not be placed in the table and is elaborately discussed in this section.

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TABLE I
SUMMARY OF DIFFERENT MR IMAGE DENOISING TECHNIQUES

Methods	Category	Merits	Demerits	Ref.
ICA based approaches	Data adaptive transform	The use of ICA algorithm eliminates the noise content in the image by decomposing the statistical independent and non-Gaussian data vectors. This approach of denoising enhances the edge sharpness while retaining the quality of the reconstructed MR image.	The sliding windowing approach increases the computational complexity. Further, the need of noise free sample data or a minimum of two images of same scene, reduces the feasibility.	[12]-[14] 2003
FT based approaches	Freq. domain	These approaches are found effective for denoising MR images with higher value of SNR at low frequency by expressing the image as the combination of true MR image with some additive noise. The denoised image is reconstructed by thresholding the noise from its detailed information content.	The denoising performance is limited to low noise images. Further, the efficiency of these methods is dependent on the threshold value selection	[15]-[17] 1991
WT based approaches	Time-Scale (wavelet) domain	This is a useful approach in representing image homogeneous regions separated by edges. The outlining ability of wavelet transform is utilized to decompose the MR image into definite sub-bands. The localization property of the approach makes it suitable for analyzing the nonstationary data points in a MR image.	The information content at the smooth edges is also eliminated during the noise removal process. Further, the method is nominal in representing the images with high dimensional singularities.	[18]-[32] 1991
CVT based approaches		This transform employs a phase space partitioning followed by Ridgelet transform, building the blocks of data in space and frequency. This opens up the possibility for analyzing an image with different block size using single transform. This makes the transform effective over the wide range of problems in analyzing the data for image denoising.	The method is not effective for the images with smooth regions. This results in curvelet artifacts. Further, reconstruction using this transform is redundant, hence slower.	[33]-[36] 2011
CNT based approaches		This transform is capable of exploring the two-dimensional geometric structure using sparse representation. The pyramidal directional filter bank structure helps in extracting contour and textural information in the image. The directional decomposition facilitates the allocation of different orientations and scaling for multi-resolution images.	The translation invariance in the reconstructed image introduces Gibbs like artifacts. Further, obtaining contours from the smooth region in the image increases the computational complexity.	[37]-[38] 2011
Smoothing Filters	Filtering methods	These filters are effective in reducing the Gaussian noise in the high frequency spectrum of an image. This reduces the noise variance.	The approach smoothens the sharp edges. This results in the loss of detailed information.	[41]-[46] 1985
Temporal Filters		These filters are more suitable for eliminating temporal variations in an image, such as: rapid variation, spin echo effects and object movement.	A major problem with this kind of filtering is that it reduces the noise as well as the image content, while spectral deviation is non-zero.	[16] 1985
AD Filters		This approach gives a single boundary solution for the complete image in spatial domain. Biased and modification of the conventional AD filters are effective in defining mathematical problems. This also gives a steady state solution.	Small features at the edge regions are eliminated due to transformed image statistics.	[47]-[53] 1992
NLM Filters		Here, the gray level similarity of a pixel value restores the redundancy in its neighborhood. The weight factor computed from the spatial similarity and mean differences is used for restoring the flat zones present in the similar regions. This also considers the geometrical configuration of a pixel in correlation with its neighboring pixels for preserving straight or curved edges.	The filters need intensive computation of the Euclidean distance among the neighboring patches which increases the execution time.	[54]-[63] 2008
Bilateral Filters		The use of photometric similarity property in bilateral filtering makes it effective in combining the gray levels to their geometrical similar neighboring pixels. The local and non-iterative approach ensures the edge preservation in the homogeneous regions of an image. The use of the perceptual metric for smoothing gray levels and preserving edges, makes it suitable for smart health care system.	The denoising and edge preservation performance is dependent on the parameter setting. It is also observed that the filtering approach is not suitable in removing noise from lower spectral regions.	[64]-[69] 2006
ML estimation based approaches	Stochastic methods	The unbiased computational approach for parameter estimation over a large number of random samples makes it suitable for denoising images with random noise. The degree of freedom for the orientation and scaling vectors are computed from the correlation of pixels in the homogeneous region. This also makes the approach effective for denoising images with small data samples.	Signal correlation in the image results in some biased data points in reconstructed image. Further, the parameter estimation of non-Gaussian data samples may be trapped in local optima.	[70]-[77] 1998
EM estimation based approaches		This parameter estimation approach is independent of the number of background pixels in the noisy image. The computation of noise variance is avoided due to the latent variables in the parameter estimation process. The computational complexity is reduced due to the iterative learning process.	The parameters may convergence to the local optima. Further, the convergence rate is slow due to the iterative learning.	[78]-[79] 2009
LMMSE estimation based approaches		The method estimates the actual value of a pixel using the local statistical information. Considering the realization of noise free signal, the LMMSE estimation provides a closed form analytical solution. This makes the approach computationally effective for analyzing complex MR data.	The parameter estimation is restricted to the samples from the local neighborhood only. Further, noise variance factor is prone to outlier due to the process of error value estimation.	[80]-[83] 2008
Bayesian estimation based approaches		This stochastic approach does not use the supporting parameters for estimating the actual information content of a data point. The higher order coherent data are computed from the noisy MR images, assuming to be Markov random fields.	The methods employ hypothesis testing rather than parameter estimation. The methods are computationally complex for the large volume of MR data.	[84]-[88] 2005
Phase Error based approaches		This approach uses a series of nonlinear filtering for the phase estimation of the noisy image. This gives MAP estimation of phase and reflectance. The phase error component in the MR image is eliminated. The restoration procedure indicates the effective preservation of edges in the MR images.	The accurate phase error estimation is highly essential. The process is also computationally complex.	[89] 2005
HPDE based approaches	PDE based methods	These filters are very effective in eliminating the oscillations at higher frequencies, while preserving edges. They offer the flexibility in implementing various functions in reducing the different noises in MR images.	The discretized application of the process converges to a constant after a few number of iterations.	[90]-[96] 2003

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TABLE II
EVALUATION INDICES USED FOR VALIDATING DENOISING TECHNIQUES

Indices	Formula	Description
Mean square Error (MSE) [120]	$MSE=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left[X_{ref}(m,n)-X_{dno}(m,n)\right]^2$	This is a commonly used distortion measure. The parameter estimates the average of the square of errors. The parameter is nonnegative and values closer to zero are better.
Peak Signal to Noise ratio (PSNR) [120]	$PSNR=10\log_{10}\frac{X_{\max}^2}{MSE}$	This is extensively used for measuring the quality of the restored image. The parameter is defined as the ratio of peak signal power to the amount of noise in the denoised MR image. A higher PSNR value indicates better denoising ability of the scheme.
Normalized Absolute Error (NAE) [121]	$NAE=\frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left\|X_{ref}(m,n)-X_{dno}(m,n)\right\|}{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left\|X_{ref}(m,n)\right\|}$	This parameter shows the error value estimated from the intensity differences. A lower value approximating zero indicates lessor error in the restored image.
Maximum Difference (MD) [121]	$MD=\max\left(\left\|X_{ref}(m,n)-X_{dno}(m,n)\right\|\right)$	The parameter is a pixel difference based measure for evaluating the error value between the reference image and the denoised image. A lower value indicates better image quality.
Structural Content (SC) [121]	$SC=\frac{\sum_{m=0}^{M-1}\sum_{n=0}^{M-1}(\mathrm{X}_{ref}^2)}{\sum_{m=0}^{M-1}\sum_{n=0}^{M-1}(\mathrm{X}_{dno}^2)}$	The parameter is a correlation-based approach for measuring the structural similarity between the reference image and the denoised image. A lower value of the index shows better preservation of image quality.
Normalized Cross Correlation (NCC) [121]	$NCC=\frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}X_{ref}(m,n)X_{dno}(m,n)}{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}X_{ref}^2(m,n)}$	This parameter is used for computing the spectral feature similarity of the restored image with the reference image. A higher value approximating 1 is better.
Structural Similarity Index (SSIM) [122]	$SSIM=\frac{(2\mu_{X_{ref}}\mu_{X_{dno}}+c_1)(2\sigma_{X_{ref}X_{dno}}+c_2)}{(\mu_{X_{ref}}^2+\mu_{X_{dno}}^2+c_1)(\sigma_{X_{ref}}^2+\sigma_{X_{dno}}^2+c_2)}$	The parameter is computed for finding the similarity between the reference image and the denoised image. Its value should be within [0, 1]. A higher value indicates better-restored image.
Quality Index based on Local Var. (QILV) [123]	$QILV=\frac{2\mu_{X_{ref}}\mu_{X_{dno}}}{\mu_{X_{ref}}^2+\mu_{X_{dno}}^2}\frac{2\sigma_{X_{ref}}\sigma_{X_{dno}}}{\sigma_{X_{ref}}^2+\sigma_{X_{dno}}^2}\frac{\sigma_{X_{ref}X_{dno}}}{\sigma_{X_{ref}}\sigma_{X_{dno}}}$	This gives a comparison of local variance distribution of the restored image with respect to the reference image. A higher index value indicates better image quality.
Image Quality Index (IQI) [124]	$IQI=\frac{\sigma_{X_{ref}X_{dno}}}{\sigma_{X_{ref}}\sigma_{X_{dno}}}\frac{2\mu_{X_{ref}}\mu_{X_{dno}}}{\mu_{X_{ref}}^2+\mu_{X_{dno}}^2}\frac{\sigma_{X_{ref}}\sigma_{X_{dno}}}{\sigma_{X_{ref}}^2+\sigma_{X_{dno}}^2}$	This parameter indicates the accuracy of denoised image due to luminance and contrast distortion. This gives a weighted mixture of visually important qualities of an image. The index value approximating 1 indicates better image quality.
Bhattacharya Coefficient (BC) [125]	$BC=\frac{1}{\sqrt{2\pi(\sigma_{X_{ref}}^2+\sigma_{X_{dno}}^2)}}\exp\left\{\frac{-(\mu_{X_{ref}}-\mu_{X_{dno}})}{2(\sigma_{X_{ref}}^2+\sigma_{X_{dno}}^2)}\right\}$	This geometric similarity measure shows the probability of misinterpreted data points in the restored image. The values closer to 1 indicates better similarity in distribution.
Mutual Information (MI) [61]	$MI=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}p_{X_{ref},X_{dno}}(m,n)\log\frac{p_{X_{ref},X_{dno}}(m,n)}{p_{X_{ref}}(m,n)p_{X_{dno}}(m,n)}$	The parameter shows the mutual dependency between the reference image and the denoised image. The higher value shows better registration of image.
Relative Contrast (RC) [28]	$RC=\frac{\left\|\mu_{X_{ref}}-\mu_{X_{dno}}\right\|}{\sqrt{\sigma_{X_{ref}}\sigma_{X_{dno}}}}$	The parameter gives a quantitative value of object to background contrast ratio relating to the residual noise. A higher value of the metric indicates better denoising performance.
Beta Metrics (BM) [47]	$BM=\frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}[X_{ref}(m,n)-\bar{X}_{ref}(m,n)]\times[X_{dno}(m,n)-\bar{X}_{dno}(m,n)]}{\sqrt{\sum_{n=0}^{N-1}[X_{ref}(m,n)-\bar{X}_{ref}(m,n)]^2\times[X_{dno}(m,n)-\bar{X}_{dno}(m,n)]^2}}$	The parameter is used for evaluating the structural preservation in the restored image. Its value lies in the range [0, 1]. A value close to 1 indicates better structural preservation.

III. EVALUATION PARAMETERS AND DATABASES

MR image denoising is a fundamental pre-processing requirement for the computerized analysis. This includes the process of estimating the noise content in an image while preserving the fine detailed information at the edges. The anatomical structure of brain is a critical issue for the processing and extracting information. As discussed in section II, there are several methods studied in the literature for denoising the brain MR images. The discussed schemes used different databases and evaluation indices for the validation. Therefore, a fair comparison among the techniques is quite difficult.

The quantitative assessment of the algorithms is carried out using different denoising and structural evaluation indices. There are a large number of evaluation indices found in the literature for validating the denoising techniques. More specifically, SNR, MSE, structural similarity (SSIM), quality index based local variance (QILV) are the most used evaluation indices. A brief explanation of many other evaluation indices is presented in Table II. The symbol X_{ref} represents the reference image and X_{dno} represents the denoised image. The other symbols used in the formulas carry the same meaning as specified in the corresponding literature.

A. Brain MR Image Databases

The performance evaluation of a denoising technique uses synthetic as well as clinical brain MR images. A large number of online databases are found that provide synthetic and clinical brain MR images. Table III gives a brief information about the publically available databases with their URLs and modalities.

1) *Synthetic image*

A synthetic MR image is produced using a computer system with advanced designing tools without an actual MRI scanner. The images are synthesized approximating the anatomical structure of the human brain using next generation techniques. It facilitates the researchers for generating a broad range of MR images with given specification. For instance, various

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modalities of brain MR images are constructed by setting the echo time and repetition time. Further, it also allows customizing the image slice thickness, noise level, intensity inhomogeneity level etc. In addition, ground truth synthetic image is also available as the reference for evaluating the performance of a denoising technique. Fig. 4 shows the example of T1-w, T2-w and PD synthetic brain MR images taken from the BrainWeb database. Each image volume contains 1mm slice thickness of size 181×217×181voxels.

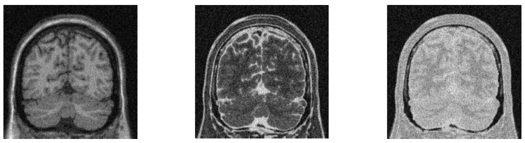

Fig. 4. Example synthetic image (with 9% noise) from BrainWeb database.

2) *Clinical image*

The performance evaluation of a medical image denoising technique also requires the use of clinical images. They are the real MR images obtained using an MR image scanner. The unavailability of a ground truth clinical image makes it difficult for validation purpose. Fig. 5 shows different modalities (T1-w, T2-w and PD) of clinical brain MR images taken from Allen brain atlas database.

Fig. 5. Example Clinical images with noise from Allen brain atlas.

TABLE III
COMMONLY USED BRAIN MR IMAGE DATABASES

Database	URL	Available image modality
Brain Web	https://brainweb.bic.mni.mcgill.ca/mri_sim.html	Synthetic brain MR images with various modalities, percentage of noise, intensity inhomogeneity and slice thickness.
IBSR	https://www.nitrc.org/projects/ibsr	Clinical brain MR images with manually guided anatomical structures. This also provides the segmented ground truth images for the evaluation purpose.
Harvard Whole Brain Atlas	https://www.med.harvard.edu/AANLIB/home.html	Clinical brain MR images containing normal and diseased brain, such as: cerebrovascular, tumors, infectious disease images, etc.
BITE: NIST Lab	http://www.nist.mni.mcgill.ca	Vivo clinical images of patients with brain tumors.
NBDC Human Data	https://humandbs.biosciencedbc.jp/data-use	Synthetic human anatomical data generated with advances in next generation sequencing for various modalities and artifacts.
Allen brain atlas	http://www.brain-map.org	Multimodal atlas of the integrated anatomic and genomic information contained brain data, such as: Under growing brain atlas, traumatic injuries, dementia and spinal cord atlas.
QTIM Lab.	http://martinos.org/qtim/miccai2013/data.html	Quantitative translational and multi-contrast MR images with infectious diseases.

IV. DISCUSSION

Denoising is an essential pre-processing requirement for all MR image diagnosis procedures. Several techniques are discussed in detail in section II. A fair comparison of the discussed techniques is quite a tedious task. This is due to the use of distinct MR images from different databases. Further, the authors have used different evaluation indices for the validation purpose. For a fair comparison among the different methods, we have used the input and the output images given in the respective papers for computing the values of evaluation metrics. The denoising results are collected from the papers with (9%-12%) of Rician noise. The aim of this study is to assimilate the recent findings and present the new aspects on MR image denoising. The survey is conducted considering more than 100 number of research articles in the last two decades. The quantitative analysis of different methods is presented in Table IV-VIII. Note that the figures with bold faces in the tables show the best in class results. The '-' in all the tables indicate unavailability of images/data in the respective papers.

Preserving the structural details is an important concern in the process of MR image denoising. In spatial domain methods, preserving the edge details still remains a challenging factor. Table IV shows a quantitative comparison of different data adaptive and non-data adaptive approaches. Here, it is observed that the CVT approaches are found to be effective in denoising brain MR images. This is evident from the best values of MSE and PSNR. This may be due to the use of local Ridgelet transform in CVT domain for eliminating Rician noise in MR images. Further, restoration of multi-scale geometry components preserves most of the structural details. The directions of edges are obtained from the orientation and anisotropy information from its multi-scale geometry. The NCC and IQI values in the table show the performance of CVT in preserving structural details, while denoising. Although, the best denoising performance is observed with the CVT domain approaches, the MSE and PSNR evaluation indices with the BM4D approach are also close to the best values. Further, the best values of SSIM, QILV and BC indicate the efficacy of the approach in preserving the structural details. This may be due to the sparse representation of data points for obtaining the optimal thresholding values and multi-dimensional block matching in denoising. Further, the improved thresholding mechanism is automatic and adaptive to the statistical characteristic of random noise in the MR data.

A comparison of different filtering methods as specified in Fig. 2 is presented in Table V. Among them, the performance of NLM based filtering approach is found to be effective. This is evident from the best values of MSE and PSNR in the table. These values are stated best in the OBNLM filtering technique. This may be due to the use of nonlocal pixel similarity in exploiting the redundant information in the image. Further, the evaluation indices NCC, BM, IQI and BC are showing the best values with this approach. This shows better preservation of image details, while denoising. Further, the evaluation indices SSIM and QILV are found to be better with the UNLM based techniques. This indicates the superiority of the approach in preserving structural details in MR images. In the meantime, their denoising performance is also closer to the NLM filtering

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TABLE IV
COMPARISON OF DIFFERENT DATA ADAPTIVE AND NON-DATA ADAPTIVE METHODS USED IN DENOISING BRAIN MR IMAGES

Approaches	MSE	PSNR	SSIM	QILV	NCC	BM	IQI	BC	MI	NAE
WT [51]	38.18	30.56	0.8459	0.8718	0.9423	0.9025	0.9234	0.8819	2.001	0.2202
SWT [102]	37.85	30.22	0.8718	0.8634	0.9275	0.9147	0.9027	0.8744	1.8742	0.1823
CVT [36]	**31.62**	**33.46**	0.9573	0.8814	**0.9752**	0.9124	**0.9441**	0.8824	1.9241	0.1927
CNT [37]	36.52	32.25	0.9438	-	0.9456	-	0.8528	-	-	-
BM3D [39]	36.24	31.28	0.9457	0.9539	0.9417	**0.9308**	0.9119	0.8751	2.0132	0.1011
BM4D [39]	35.23	32.32	**0.9759**	**0.9872**	0.9246	0.9142	0.9301	**0.9078**	**2.2215**	**0.0857**
ICA [13]	30.34	25.82	0.9121	-	-	-	-	-	-	0.1731

TABLE V
COMPARISON OF DIFFERENT FILTERING METHODS USED IN DENOISING BRAIN MR IMAGES

Approaches	MSE	PSNR	SSIM	QILV	NCC	BM	IQI	BC	MI	NAE
WF [91]	87.91	25.41	0.7592	0.7839	0.7028	0.7541	0.7423	0.7784	1.3451	0.1556
MF [91]	53.71	25.99	0.7548	0.7823	0.7041	0.7624	0.7514	0.7851	1.4214	0.2673
AMF [91]	44.33	27.18	0.7941	0.7951	0.7124	0.7628	0.7589	0.7891	1.5122	0.2475
AD [51]	45.18	27.51	0.8027	0.8164	0.6841	0.7527	0.7432	0.7954	2.0241	0.2054
NDAD [51]	44.24	30.8	0.8432	0.8893	0.8003	0.7788	0.7834	0.8624	**2.7201**	0.1151
NLM [57]	40.21	32.04	0.8531	0.8956	0.9127	0.6899	0.8847	**0.8834**	1.9887	0.1034
OBNLM [58]	**31.35**	**35.64**	0.9064	0.9112	**0.9236**	**0.8314**	**0.8926**	0.8756	2.2141	**0.0998**
UNLM [60]	41.02	33.18	**0.9131**	**0.9156**	0.9221	0.7103	0.8882	0.8567	2.2582	0.1051
BF [67]	44.53	28.77	0.8683	0.8966	0.8284	0.7485	0.7772	0.6959	2.1317	0.1763
SBF [67]	51.31	28.45	0.8323	0.8695	0.8448	0.7721	0.8026	0.7264	2.0605	0.2041
MRBF [67]	41.01	31.31	0.9008	0.8951	0.8754	0.8062	0.6995	0.8091	2.1846	0.1292
RSBF [67]	42.44	31.13	0.8956	0.9092	0.8879	0.8021	0.7967	0.7628	2.0111	0.1955
TF [68]	43.12	29.35	0.8547	-	-	-	-	-	-	-

TABLE VI
COMPARISON OF DIFFERENT STOCHASTIC METHODS USED IN DENOISING BRAIN MR IMAGES

Approaches	MSE	PSNR	SSIM	QILV	NCC	BM	IQI	BC	MI	NAE
ML [81]	51.25	28.27	0.8681	0.8019	0.8117	0.7541	0.7835	0.7547	1.9128	0.1339
LML [83]	51.53	28.48	0.9101	0.8126	0.8234	0.7624	0.7935	0.8964	2.0124	0.1221
NLML [83]	40.27	30.17	**0.9145**	0.8261	0.8361	0.7721	0.8127	**0.9021**	2.0342	0.1117
EM [81]	44.11	22.18	0.8615	0.6516	0.8457	0.7442	0.8364	0.8842	1.9872	0.1242
LMMSE [81]	38.61	29.17	0.8208	0.8325	0.8528	0.7029	0.8968	0.8479	2.1501	0.1037
ALMMSE [83]	**29.67**	**38.26**	0.8949	0.8674	**0.8625**	0.7389	**0.9168**	0.8937	2.2379	**0.1014**
RLMMSE [83]	44.05	32.42	0.9032	**0.8793**	0.8248	**0.7841**	0.8081	0.8651	**2.4431**	0.1121

TABLE VII
COMPARISON OF DIFFERENT PDE BASED METHODS USED IN DENOISING BRAIN MR IMAGES

Approaches	MSE	PSNR	SSIM	QILV	NCC	BM	IQI	BC	MI	NAE
PDE [90]	42.52	28.35	0.8521	0.7958	0.8035	0.7851	0.8015	0.8534	1.8954	0.1247
Modified PDE [90]	40.12	30.25	**0.8615**	**0.8392**	0.8237	**0.8102**	**0.8446**	**0.8872**	1.6742	0.0987
Adaptive PDE [95]	**34.25**	**32.87**	0.8787	0.8364	**0.8482**	0.7557	0.8115	0.8641	**2.1002**	**0.0627**

TABLE VIII
COMPARISON OF DIFFERENT HYBRID TECHNIQUES

Methods	MSE	PSNR	SSIM
NLM-PCA [107]	32.26	31.71	0.8927
NLM-FC [97]	29.14	35.90	0.8936
UNLM-PCA [106]	**28.61**	**38.64**	**0.9601**
WT-CVT [102]	33.54	30.20	0.8869
WT-BF [27]	62.26	26.53	0.9080
BF-GA [109]	33.59	30.13	0.8833
LMMSE-PCA [108]	30.28	34.14	0.9190

based approaches. They are specifically effective in eliminating blocky effects. The lowest value of NAE with this approach shows the least possible erroneous data in the restored image. However, their performance is not consistent with the multidimensional and multiresolution brain MR images. BF is found to be effective for these specific applications. The use of geometric and photometric features in the denoising process, makes it suitable for multidimensional and multiresolution MR images. The non-iterative mechanism makes it computationally effective in comparison to others. The best values of MI and NAE indicate the better registration of image details with lesser error. In addition to the discussed schemes, there are some hybrid approaches studied in the literature.

From the literature, it is observed that the filtering methods are effective in denoising Gaussian noise in MR images. However, they introduce blurring in the edge regions, i.e. small details in the edge regions are eliminated. Table VI shows a comparative analysis of different stochastic methods as specified in Fig. 3. The ALMMSE estimation based approaches are found to be useful in denoising brain MR images. This is evident from the best in class values of MSE and PSNR in the table. This may be due to the inclusion of spatial information with acquired natural redundancy in computing the minimum MSE values. The adaptive nature of the approach towards the variation of local noise in the MR images makes it useful in preserving the structural details while denoising. This is evident from the best values of NCC and IQI parameters in the table. Further, the RLMMSE estimation based approaches effectively preserves the structural details. This is identified from the best values of QILV and BM parameters in the table. This may be due to the inclusion of diffusion weighted nonlinear information into the computation of minimum MSE value.

However, the approach is precisely effective for denoising Rician noise in MR images. The optimum value of MI with this approach indicates better registration of the image details. In combination with the LMMSE based approaches, the NLML estimation based approaches are also found to be effective for preserving the structural details. This is marked from the best values of SSIM and BC evaluation indices in Table VI. From the literature, it is also observed that the stochastic approaches in transform domain are effective in denoising non-stationary Rician noise in the clinical MR images. However, the computational efficiency is dependent on the population of MR data.

A comparison of different PDE based MR image denoising methods as is shown in Table VII. From the study, it is observed that the PDE based methods are preserving edge details and the tissue regions in the non-homogeneous regions. However, the methods denoise the MR images at the cost of computational complexity. The adaptive PDE based method is found to be useful in denoising. This is evident from the best in class values of MSE and PSNR in the table. This may be due to the adaptive nature of the method towards the noise variation in the MR image. Further, modified PDE based method is effectively preserves the structural details. This is identified from the best values of SSIM, QILV, BM, IQI and BC parameters in the table. This may be due to the inclusion of diffusion weight in computation of nonlinear information. However, the method is used in denoising non-Gaussian noise with low SNR. The values of MI indicate better registration of the image details. From the literature, it is also observed that the PDE based methods are effective in denoising non-Gaussian noise in the MR images. However, the performance of the methods is limited due to their computational complexity.

Table VIII presents the performance comparison of some of the hybrid approaches used in denoising brain MR images. The best values in the table show the hybridization of UNLM and PCA for denoising as well as preserving structural details. This may be due to the nonlocal PCA based thresholding of the images by automatic estimation of spatially varying local noise. Further, the use of UNLM filter makes it rotationally invariant. However, the efficacy of a particular method is dependent on the choice of imaging modality, level of noise and filter parameters. This opens the scope of research in this particular area.

From the survey, it is found that the direction of the research is advancing towards automation of denoising techniques. Further, the recent trends show the use of artificial intelligence to make the schemes more feasible for clinical applications. A specific denoising scheme can be applicable for a particular modality of MR image with given noise. For instance, filtering methods are the simplest in implementation. They effectively denoise Gaussian noise in MR images. However, the performance is limited due to loss of structural details and creating blurred regions. Non-data adaptive approaches are providing better results for Rician and Rayleigh distributed noise in MR images. Stochastic methods are effective in denoising Gaussian and non-Gaussian noise in complex and real MR images. However, the computational efficiency is dependent on the population of MR data. PDE based methods are found to be effective in denoising non-Gaussian noise in the MR images. Hybridization of stochastic methods in transform domain or use of optimization tools or learning based methods is found to be effective for denoising non-stationary Rician noise in the clinical MR images. In MR imaging modalities, T1-w images are preferably chosen for discriminating inter-tissue regions, whereas T2-w image are chosen for intra-tissue regions. Now a day, there are various MR modalities (FLAIR, dMRI) developed for more feasible clinical usage.

V. CONCLUSION

This paper provides a framework for categorizing the state-of-the-art algorithms used in MR image denoising. The denoising techniques are grouped into spatial and transform domain based on the image model used for medical image processing. Further, they are categorized as filtering methods, stochastic methods, partial differential equation (PDE) based methods and hybrid methods. The proposed categorization is simplifying the complex system, helps in problem formulation and critical experimentation. A quantitative analysis is carried out using a wide range of evaluation indices, showing denoising and structural similarity in the restored images. This suggests the appropriate evaluation indices to be used in MR image denoising and the best method to denoise MR image with given noise. The findings of the study are – 1) Filtering methods are simpler and effective for eliminating Gaussian noise from the homogeneous regions. The potential drawback of the method is that they eliminate the small structures and the edge details by blurring the non-homogeneous regions. 2) Wavelet based transform domain approaches combined with stochastic methods are found to be effective in denoising and preserving edge details in the complex MR images. It is noteworthy to mention here that none of the methods discussed above is effective individually in solving the problem on hand. However, a justifiable combination of any of the approaches may bring fantastic results in denoising MR images. 3) Hybridizing the above two with stochastic approaches, optimization tools or the learning based methods gives better denoising performance. From this survey, the researchers may get knowledge about the most appropriate denoising technique for any specific MR image. The survey also highlights the challenges faced while using different denoising schemes and the inherent problems with various imaging modalities. In addition, many possible modifications are marked as the future direction for improving the performance of the existing MR image denoising techniques.

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