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Rough computing — A review of abstraction, hybridization and extent of applications

D.P. Acharjya [a,*], Ajith Abraham [b]

[a] School of Computer Science and Engineering, VIT, Vellore 632014, India
[b] Machine Intelligence Research Labs, 11, 3rd Street NW, Auburn, WA 98071, USA

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ABSTRACT

The rapid growth of information and communication technology captured common man and various organizations and influenced each individual's life, work, and study. It leads to a data explosion. It has no utility without any analysis and leads to many analytical techniques. The prime objective of these techniques is to derive some useful knowledge. However, the transformation of data into knowledge is not easy because of many reasons, such as disorganized, incomplete, uncertainties, etc. Furthermore, analyzing uncertainties present in data is not a straight forward task. Many different models, like fuzzy sets, rough sets, soft sets, neural networks, generalizations, and hybrid models obtained by combining two or more of these models, have been fruitful in representing knowledge. To this end, this paper identifies the conventionally used rough computing techniques and discusses their concepts, developments, abstraction, hybridization, and scope of applications.

1. Introduction

The rapid growth of information and communication technology captured common man, various organizations, and influences each individual's life, work, and study. It leads to a data explosion. Data are of no use unless we derive some knowledge from it, and it leads to knowledge discovery, knowledge representation, and information retrieval. Several conventional techniques are available to analyze these data. Nevertheless, these conventional techniques fail to analyze these data if the data contains uncertainties. Further to analyze these uncertainties, many intelligent techniques, such as fuzzy set (Zadeh, 1965), L fuzzy set (Goguen, 1967), rough set (Pawlak, 1982), intuitionistic fuzzy set (Atanassov, 1986), two-fold fuzzy set (Dubois and Prade, 1987), soft set (Molodtsov, 1999), neural network (Hansen and Salamon, 1990), and hybrid models combining two or more of these models are developed. Among these methods, the rough set is relatively new in knowledge extraction while handling uncertainties and impreciseness. It has inspired many researchers to carry out their research in different domains that deal with imperfect knowledge.

From a philosophical point of view, the rough set is a new approach to deal with vagueness and uncertainty. The algebraic properties of the rough set were studied extensively by many researchers (Grzymala-Busse, 1988; Iwinski, 1988; Novotny and Pawlak, 1985a,b; Nieminen, 1988; Obtulowicz, 1987; Davvaz and Mahdavipour, 2008). Besides, algebraic semantics are studied, and more generalized properties are explored (Pagliani and Chakraborty, 2007; Mani, 2009; Banerjee and Chakraborty, 1994; Bunder et al., 2008). These developments are applied in diversified fields such as machine learning, knowledge acquisition, decision analysis, knowledge discovery from databases, expert systems, inductive reasoning, pattern recognition, and many real-life problems. The equivalence relation of the rough set is generalized to many extents. To name a few, the equivalence relation is generalized to fuzzy equivalence relation, fuzzy proximity relation, intuitionistic fuzzy proximity relation, covering, and binary relation. It is challenging to track these abstractions due to fast development in this domain. Additionally, the rough set is hybridized with many other theories to give promising results. A few are hybridized with the neural network, genetic algorithm, formal concept analysis, support vector machine, and bioinspired computing. However, no comprehensive study has been made to explore the potential of various abstractions of rough set across different real-life applications.

All over the world, rough computing is widely researched. The original idea of this model is the classification of an information system based on an indiscernibility relation (Pawlak, 1984, 1981). Besides, it has the consideration that all perception is subject to granularity (Yao, 2001), Boolean reasoning (Pawlak and Skowron, 2007a), and classification of human intelligence (Pawlak and Skowron, 2007c,b). The approximation spaces of The rough set refer to multiple memberships, whereas the fuzzy set refers to partial membership. The expeditious growth of these two theories forms the beginning of soft computing in artificial intelligence. However, a rough set is unique because it uses

* Corresponding author.
E-mail address: debiprasannaacharjya@vit.ac.in (D.P. Acharjya).

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only the information given by the operationalized data. In fact, few researchers are familiar with these developments. Simultaneously, many abstractions and hybridization models are approaching rapidly and gaining their prominence. Nevertheless, these models are not explored across various domains due to lake of visibility among researchers. Many times researchers force-fit a particular model to their problem instead of analyzing all the developed models. Therefore, it is essential to put forth all the models so that a researcher can easily identify a suitable model for their study.

With this objective, this paper discusses rough set, its various variations, hybridizations with other concepts and notions, its applications in science and technology. The paper mainly emphasizes concepts and notions of rough computing and its hybridizations. The rest of the paper is organized as follows. Following the introduction, the research methodology is discussed in Section 2. The abstraction of rough computing is presented in Section 3, which presents various types of rough sets. Section 4 presents hybridized rough computing. Software libraries available for rough set is discussed in Section 5. This section highlights the hybridization of the rough set with other concepts. Various applications of rough computing are presented in Section 6. The article is concluded in Section 7.

2. Research methodology

The evolution of information and communication technology brought a rapid change in the way the data are collected, and the decisions are taken. Data are generally collected from various sources and are represented in the form of an information system. An information system typically contains a countable set of objects characterized by a set of attributes. Such an information system may conveniently be described in a tabular form in which rows represent objects and columns as attributes. Each cell of an information system provides the attribute information of a particular object. Additionally, the information system may be disorganized, incomplete, imprecise, and may contain uncertainties. The prime objective of inductive learning is to learn the knowledge for classification. For classification, the rough set theory (Pawlak, 1991) is a relatively new knowledge discovery tool that has inspired many researchers to carry out their research in different domains. The main feature of the rough set data analysis tool is non-invasive, and the ability to handle qualitative data. It fits into most real-life applications nicely. So far, this theory has achieved many interesting and promising results (Ziarko, 1993; Slowinski and Zopounidis, 1995; Zhong, 2001; Shen et al., 2000; Polkowski, 2013) and has its importance as a mathematical tool.

Rough set data analysis is based on indiscernibility relation defined over a single universe. However, in many real-life problems, it is challenging to establish an indiscernibility relation between objects. Additionally, there may be an information system that establishes a relation between the two universes. Keeping view of all these, the research has been carried out in three phases. In the first phase, we discuss in this article on some extensions of rough sets defined through less stringent relations than an indiscernibility relation. Additionally, we discuss extensions of the rough set on two universal sets. Further, the rough set is hybridized with many other concepts such as neural networks, formal concept analysis, bio-inspired algorithms, etc. We discuss these hybridizations in the second phase of research work. Indeed, several applications are being carried out by various researchers. The third phase of research highlights those real-life applications.

Identifying variations of the rough set and its hybridization with other concepts and its applications was a significant challenge as many of these are published in conference proceedings and book chapters. Simultaneously, it is not wise to ignore them. The prime reason behind this is the time taken by the peer-reviewed journals. Therefore, we have restricted our search to Scopus indexed journals, conferences, and Google Scholar directory to identify the research done in this direction. Additionally, keywords like the rough set, lower and upper approximation, rough computing, and intelligent computing are considered. The prime objective was to search the current developments and applications of the rough set. Further, we extended the search for finding the concepts, notions, and variations pertaining to rough set. A review of this literature helps us to find the potential scope of applications for some of these rough set variations.

A narrative approach with a brief mathematical explanation is followed in the first phase of research, whereas the narrative review approach is followed in the second phase. The current research is mainly focused on more excellent coverage of studies. Further, many of such work is not available freely due to various indexing rules, and in such cases, we have restricted to abstract, application, and conclusion. Nevertheless, an intensive review has been done, there may be a chance of leaving out some recent developments. It is another limitation of our study.

3. Abstraction of rough computing

The standard set has been extended in many directions. Fuzzy set of Zadeh (Zadeh, 1965) and the rough set of Pawlak (Pawlak, 1982) being noteworthy among them. The rough set captures indiscernibility among objects. The indiscernibility among objects is established using equivalence relations. However, in many real-life problems, establishing an equivalence relation among objects is very difficult. It leads to abstraction in rough computing. The equivalence relation has been changed to fuzzy equivalence relation, fuzzy proximity relation, intuitionistic fuzzy proximity relation, covering based relation, dominance relation, to name a few. All these variations lead to different types of rough sets. Similarly, an equivalence relation is generalized to binary relation, fuzzy relation, intuitionistic fuzzy relation, and the concept of different types of rough set over two universes is defined. In the following subsections, we briefly discuss the notions and concepts of these abstractions of the rough set.

3.1. Rough set

Rough set (RS) of Pawlak is a mathematical tool to deal with imprecision, incomplete, and inconsistent data present in an information system (Pagliani and Chakraborty, 2007). The approximation of sets, reduction of superfluous attributes, knowledge representation, classification, and rule generation is the prime objective of rough set (Chang et al., 1999; Guo and Chankong, 2002; Hassanien, 2004). The basic idea is the approximation of a set by a pair of sets concerning some imprecise information that captures the indiscernibility of objects in an information system (Pawlak and Skowron, 2007a,c,b).

An information system IS is defined as 4-tuple $IS = (Q, P, V, f)$, where $Q = \{q_1, q_2, q_3, \ldots, q_n\}$ is the finite set of objects pertaining to the problem under study, $P = \{p_1, p_2, p_3, \ldots, p_m\}$ is the set of attributes or parameters, V is the union of all parameter values such that $V = \cup V_p$, $p \in P$, and f is the information function $(Q \times P) \to V$. The information system is said to be a decision system, if $P = (P_c \cup P_d)$ and $(P_c \cap P_d = \phi)$, where P_c is the set of conditional attributes and P_d is the set of decisions. Given a subset of parameters, $B \subseteq P$, an indiscernibility relation $Ind(B)$ is defined as $Ind(B) = \{(q_i, q_j) : f(q_1, p_k) = f(q_j, p_k)\}$ for all $p_k \in B$, where $q_i, q_j \in Q$. This indiscernibility relation in deed generates the equivalence class or partitions the objects into various classes Q/B. Given a target set $X \subseteq Q$, the B-lower and B-upper approximations is defined as below.

$$\underline{B}X = \cup\{Y \in Q/B : Y \subseteq X\} \tag{1}$$

$$\overline{B}X = \cup\{Y \in Q/B : Y \cap X \neq \phi\} \tag{2}$$

It leads to two cases such as $\underline{B}X = \overline{B}X$ or $\underline{B}X \neq \overline{B}X$. In the former case, the target set X is a classical set whereas in later case it is a rough set. The objects which lies in $(\overline{B}X - \underline{B}X)$ are called as boundary line objects and is denoted as $BL_B(X)$. The objects belonging to $\underline{B}X$ are

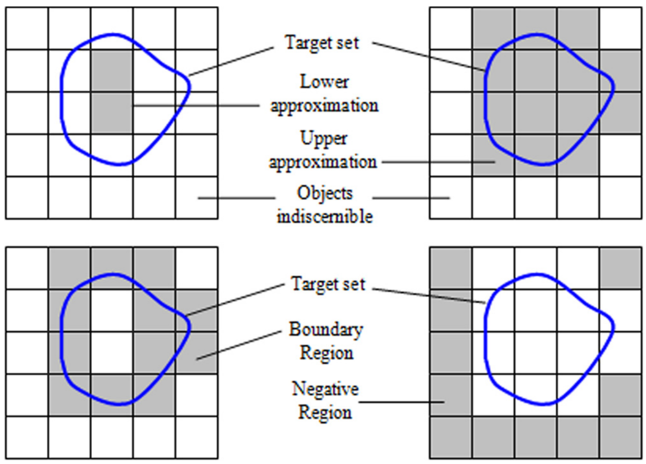

Fig. 1. Lower, upper, boundary and negative region of rough set representation.

certain whereas objects belonging to $BL_B(X)$ are uncertain. Objects belonging to $NG_B(X) = (Q - \overline{B}X)$ is known as negative region. In such cases objects do not belong to target set X. The roughness of target set X with respect to B is defined as $R_o(X)$, where

$$R_o(X) = 1 - \frac{|\underline{B}X|}{|\overline{B}X|} \tag{3}$$

It is assumed that the target set $X \neq \phi$ and $|\cdot|$ represents the cardinality of the countable set. If X is an empty set, then we have $R_o(X) = 0$. Similarly if the target set is a crisp set, then the lower and upper approximation are equal and hence $R_o(X) = 0$. The following Fig. 1 depicts the lower, upper, boundary and negative region of a rough set.

3.2. Fuzzy rough set

Pawlak's rough set works well for the categorical information system, where indiscernibility relation plays a significant role. However, in many real-life problems, the information system is quantitative rather than qualitative. Hence to apply rough set theory, the quantitative information system is to be reduced to a qualitative information system by grouping the quantitative attribute values to qualitative. However, the limitation is that there may be a loss of information. To overcome this limitation, the concept of a fuzzy rough set (FRS) was introduced. The mathematical viewpoint of this concept is fuzzy equivalence relation rather than an indiscernibility relation. The notions and concepts of a fuzzy rough set, as introduced by Dubois and Prade, are briefly defined (Dubois and Prade, 1990).

Let $Q = \{q_1, q_2, \dots, q_n\}$ is the finite set of objects of the universe, $B \subseteq P = \{p_1, p_2, \dots, p_m\}$ is the set of parameters. Let $p_i \in B$. The fuzzy equivalence class induced by p_i is denoted as $Q/Ind\{p_i\}$. Let us assume $Q/Ind\{p_i\} = \{Y_{p_i}, Z_{p_i}\}$. It means that the attribute p_i generates two fuzzy sets Y_{p_i} and Z_{p_i}. If $p_i, p_j \in P$, and $Q/Ind\{p_j\} = \{Y_{p_j}, Z_{p_j}\}$, then $Q/Ind\{p_i, p_j\} = \{Y_{p_i} \cap Y_{p_j}, Y_{p_i} \cap Z_{p_j}, Z_{p_i} \cap Y_{p_j}, Z_{p_i} \cap Z_{p_j}\}$.

Let $Q/B = \{C_1, C_2, C_3, \dots, C_k\}$ be the fuzzy equivalence class generated on employing the parameters B. Let the target fuzzy set be X. The fuzzy B-lower ($\mu_{\underline{B}X}(q)$) and fuzzy B-upper ($\mu_{\overline{B}X}(q)$) approximations are interpreted with membership of indipendent element $q \in Q$. It is defined as:

$$\mu_{\underline{B}X}(q) = sup_{C \in Q/B} \left(\mu_C(q) \wedge inf_{r \in Q}(\{1 - \mu_C(r)\} \vee \mu_X(r)) \right) \tag{4}$$

$$\mu_{\overline{B}X}(q) = sup_{C \in Q/B} \left(\mu_C(q) \wedge sup_{r \in Q}(\mu_C(r) \vee \mu_X(r)) \right) \tag{5}$$

The order pair $(\underline{B}X, \overline{B}X)$ is known as a FRS, where the B-lower and B-upper approximation of individual element of X is defined using Eqs. (4) and (5) respectively. Besides, the membership of an element $q \in Q$ belonging to such an equivalence class $C_i \in Q/B$, $i = 1, 2, \dots, k$ is calculated by applying Eq. (6).

$$\mu_{\{C_1 \cap C_2 \cap \dots \cap C_k\}}(u) = \mu_{C_1}(u) \wedge \mu_{C_2}(u) \wedge \dots \wedge \mu_{C_k}(u) \tag{6}$$

The significant advantage of FRS is that it works directly with the quantitative information system. Besides, the degree of dependency in the case of the rough set is 1, if there is no uncertainty. The degree of dependency is approximately equal to 1 in FRS, even if there is no uncertainty. Further, FRS is generalized, and a generalized fuzzy rough set is introduced (Pei, 2005). Besides, a comparison of fuzzy rough sets is also carried out in the literature (Radzikowska and Kerre, 2002).

3.3. Probabilistic rough set

Crisp rough set introduced by Pawlak (Pawlak, 1982) has certain limitation. According to traditional rough set, in lower approximation, the equivalence class is a subset of target set. Similarly, in upper approximation, the equivalence class has a non empty intersection with the target set. It indicates that, crisp rough set fails to handle uncertainty and lacking in handling error tolerance. To overcome this limitation, the concept of probabilistic rough set (PRS) is introduced (Yao, 2008; Wong and Ziarko, 1987). In PRS, the rough set approximations are defined with the help of conditional probabilities. Let $X \subseteq Q$ be the target set, and $B \subseteq P$ be the set of parameters. The conditional probability of X, that the object $q \in X$ be in the class $[q]$ is defined as $Pr(X|[q]) = \frac{|X \cap [q]|}{|[q]|}$, where $|[q]|$ refers to the cardinality of $[q]$ (Ziarko, 2008). Further, the lower and upper approximation of PRS is defined in Eqs. (7) and (8) respectively.

$$\underline{B}X = \{q \in Q : Pr(X|[q]) = 1\} \tag{7}$$

$$\overline{B}X = \{q \in Q : Pr(X|[q]) \neq 0\} \tag{8}$$

It means that the objects belonging to negative region is defined as $NG_B(X) = \{q \in Q : Pr(X|[q]) = 0\}$ and the boundary line objects is defined as $BL_B(X) = \{q \in Q : 0 < Pr(X|[q]) < 1\}$. The objects belonging to $\underline{B}X = \{q \in Q : Pr(X|[q]) = 1\}$ is termed as core of the information system. Further, PRS has extended to α-PRS, where $\alpha \in [0,1]$ is the degree of belongingness (Yao, 2008). In such cases, the lower and upper approximations are defined as below. The lower and upper approximations are dual to each other when $\alpha = 0.5$ and it refers to maximum uncertainty (Liu et al., 2011; Wei and Zhang, 2004).

$$\underline{B}_\alpha X = \{q \in Q : Pr(X|[q]) > \alpha\} \tag{9}$$

$$\overline{B}_\alpha X = \{q \in Q : Pr(X|[q]) \geq \alpha\} \tag{10}$$

3.4. Decision theoretic rough set

The traditional rough set theory make acceptance and rejection of decisions accurately. However, in PRS the acceptance and rejection of decisions is not handled accurately because of tolerance inaccuracy in lower and upper approximations. To overcome this limitation, the concept of decision theoretic rough set (DTRS) is introduced as an extension of PRS. The PRS discussed in the previous section supports only two way classification. In such cases, the object either belongs to the set or its complement. To overcome this limitation of PRS, the concept of two probabilistic threshold values is introduced in DTRS. Therefore in this case, instead of 0 and 1, two probabilistic threshold values α and β are introduced, such that $0 \leq \beta < \alpha \leq 1$. The lower and upper approximations of DTRS are defined as in Eqs. (11) and (12) respectively (Yao, 2004).

$$\underline{B}_{(\alpha,\beta)} X = \{q \in Q : Pr(X|[q]) \geq \alpha\} \tag{11}$$

$$\overline{B}_{(\alpha,\beta)} X = \{q \in Q : Pr(X|[q]) > \beta\} \tag{12}$$

It means that the objects belonging to negative region is defined as $NG_B(X) = \{q \in Q : Pr(X|[q]) \leq \beta\}$ and the boundary line objects is defined as $BL_B(X) = \{q \in Q : \beta < Pr(X|[q]) < \alpha\}$. It indicates that, the interval $[0,1]$ of probability leads to three different regions, such as acceptance region, deferment region, and rejection region. The acceptance region refers to $[\alpha,1]$, deferment region refers to (α,β), whereas rejection region refers to $[0,\beta]$. Besides, DTRS reduces to traditional rough set, when $\alpha = 1$, and $\beta = 0$ (Yao and Wong, 1992).

3.5. Variable precision rough set

Crisp rough set model has certain limitations though it classify an information system accurately. The perfect classification is achieved if the information system contains no noisy data. But, data are generated at a greater speed and often comes with noisy data. The classification using rough set in case of noisy data leads to a limitation. To overcome this limitation, the concept of variable precision rough set (VPRS) is introduced as an extension of DTRS. The prime objective of VPRS is to introduce the relative degree of misclassification (Ziarko, 1993). Let $c(X,Y)$ be the relative degree of misclassification of X with respect to Y. It is defined as in Eq. (13).

$$c(X,Y) = \begin{cases} 1 - \frac{|X\cap Y|}{|X|} & \text{if} \quad |X| > 0 \\ 0 & \text{if} \quad |X| = 0 \end{cases} \tag{13}$$

The corresponding lower and upper approximation of VPRS is defined in Eqs. (14) and (15) respectively, where β is the threshold of tolerance error.

$$\underline{B}_\beta X = \cup\{Y \in Q/B : c(Y,X) \le \beta\} \tag{14}$$

$$\overline{B}_\beta X = \cup\{Y \in Q/B : c(Y,X) < 1-\beta\} \tag{15}$$

It means that the objects belonging to negative region is defined as $NG_B(X) = \cup\{Y \in Q/B : c(Y,X) \ge 1-\beta\}$ and the boundary line objects is defined as $BL_B(X) = \cup\{Y \in Q/B : \beta < c(Y,X) < 1-\beta\}$. It indicates that, $\underline{B}_\beta X$ refers to the set of all those elements of the universal set U that are classified into X with the tolerance error not greater than β. Similarly, $NG_B(X)$ is defined as the complement of $\overline{B}_\beta X$ (Wang and Zhou, 2009).

3.6. Rough set on fuzzy approximation space

The basic idea of rough sets, introduced by Pawlak, depends upon the notion of equivalence relations defined over a universe. However, equivalence relations in real-life situations are relatively rare in practice. Therefore, efforts have been made to make the relations less significant, and the concept of fuzzy proximity relations on Q is introduced. The concept of fuzzy approximation space, which depends upon a fuzzy proximity relation defined on a set Q, is a generalization of the concept of the knowledge base. Therefore, rough sets on fuzzy approximation spaces (RSFAS) extend the concept of rough sets on knowledge bases, as discussed by Acharjya and Tripathy (Acharjya and Tripathy, 2008). We unfold the background of this article in this section by presenting the fundamental concepts, notations, and results on rough sets on fuzzy approximation spaces, and these are all bases our discussion starts from.

Let Q be a universe. We define a fuzzy relation on Q as a fuzzy subset of $(Q\times Q)$. A fuzzy relation R on Q is said to be a fuzzy proximity relation if $\mu_R(q_i,q_i) = 1$ and $\mu_R(q_i,q_j) = \mu_R(q_j,q_i)$ for $q_i,q_j \in Q$. We say that two elements q_i and q_j are α-similar, $\alpha \in [0,1]$, with respect to R if $(q_i,q_j) \in R_\alpha$ or $\mu_R(q_i,q_j) \ge \alpha$. We write it as $q_i R_\alpha q_j$. Two elements q_i and q_j are said to be α-identical with respect to R if either q_i is α-similar to q_j or q_i is transitively α-similar to q_j with respect to R, i.e., there exists a sequence $q_1, q_2, \dots, q_n \in Q$ such that $q_i R q_1$, $q_1 R q_2$, $q_2 R q_3$, $\cdots$, $q_n R q_j$. If q_i and q_j are α-identical with respect to R, then we write $q_i R(\alpha) q_j$, where the relation R for each fixed $\alpha \in [0,1]$ is an equivalence relation on Q. The pair (Q,R) is called a fuzzy approximation space. For any $\alpha \in [0,1]$, we denote by R^*_α, the set of all almost equivalence classes of $R(\alpha)$. Also, we call $(Q,R(\alpha))$, the generated approximation space associated with R and α.

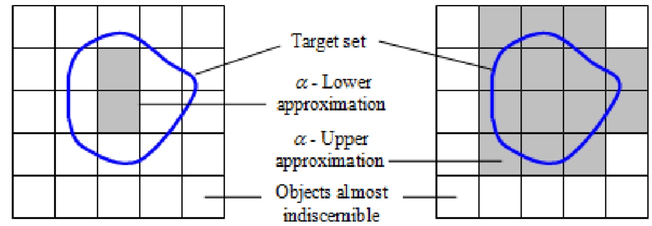

Fig. 2. Lower and upper approximation of rough set on fuzzy approximation space.

Let (Q,R) be a fuzzy approximation space and let $X \subseteq Q$ be the target set. Then the rough set of X in $(Q,R(\alpha))$ is denoted by $(\underline{R}_\alpha X, \overline{R}_\alpha X)$, where $\underline{R}_\alpha X$ is the α-lower approximation of X whereas $\overline{R}_\alpha X$ is the α-upper approximation of X. It is defined as:

$$\underline{R}_\alpha X = \cup\{Y \in R^*_\alpha : Y \subseteq X\} \tag{16}$$

$$\overline{R}_\alpha X = \cup\{Y \in R^*_\alpha : Y \cap X \ne \phi\} \tag{17}$$

The objects belonging to equivalence classes obtained through this way are almost identical or α-identical. On considering α as 100%, i.e., $\alpha = 1$; rough set on fuzzy approximation space reduces to rough set. The target set X is said to be α-crisp if $\underline{R}_\alpha X = \overline{R}_\alpha X$. Similarly, the target set X is said to be α-rough if $\underline{R}_\alpha X \ne \overline{R}_\alpha X$. The α-boundary region of X is given as $BL_{R_\alpha}(X) = \overline{R}_\alpha X - \underline{R}_\alpha X$. The objects belonging to $\underline{R}_\alpha X$ are α-certain whereas objects belonging to $BL_{R_\alpha}(X)$ are uncertain. Objects belonging to $NG_{R_\alpha}(X) = (Q - \overline{R}_\alpha X)$ is known as negative region. Fig. 2 depicts the α-lower and α-upper approximation of rough set on fuzzy approximation space.

De (De, 2004) has studied many properties of α-lower and α-upper approximations. Like the rough set, rough set on fuzzy approximation space also has four types. Characterization of basic operations such as union, intersection on types of rough sets over fuzzy approximation space is also discussed in the literature (Acharjya and Tripathy, 2008). The several ambiguity cases that may arise while union and intersection are discussed thoroughly. These are useful while studying relational database systems.

3.7. Rough set on intuitionistic fuzzy approximation space

The fuzzy set was introduced to process vagueness and uncertainties (Zadeh, 1965). Nevertheless, it has certain limitations in choosing membership functions though it has wide acceptability today. Further, it has extended to many directions, such as twofold fuzzy sets (Dubois and Prade, 1987), L-fuzzy set (Goguen, 1967), toll sets (Dubois and Prade, 1993), and intuitionistic fuzzy sets (Atanassov, 1986). However, the intuitionistic fuzzy set is much useful and applicable in many real-life problems because of the presence of hesitation. In a fuzzy set, if μ be the degree of membership of an element q, then the degree of non-membership of q is calculated using a mathematical formula with the assumption that full part of the degree of membership is determinism and in-deterministic part is zero. At the same time, the intuitionistic fuzzy set reduces to a fuzzy set if the in-deterministic part is zero. Thus, the intuitionistic fuzzy set is a generalized and better model over the fuzzy set model. Thus, rough sets on intuitionistic fuzzy approximation spaces (RSIFAS) is a generalized and better model then RSFAS. In this article, the definitions, notations, and results on RSIFAS are briefly presented (Tripathy, 2006). The basic concepts of RSIFAS use the standard notation μ for membership and ν for non-membership.

Let Q be an universal set of objects. An intuitionistic fuzzy relation R on a universal set Q is an intuitionistic fuzzy set defined on $(Q \times Q)$. An intuitionistic fuzzy relation R on Q is said to be an intuitionistic fuzzy proximity relation if and only if $\mu_R(q_i,q_i) = 1$, $\nu_R(q_i,q_i) = 0$ for all $q_i \in Q$; and $\mu_R(q_i,q_j) = \mu_R(q_j,q_i)$, $\nu_R(q_i,q_j) = \nu_R(q_j,q_i)$ for all $q_i,q_j \in Q$. Let R be an intuitionistic fuzzy proximity relation on Q. Then for any

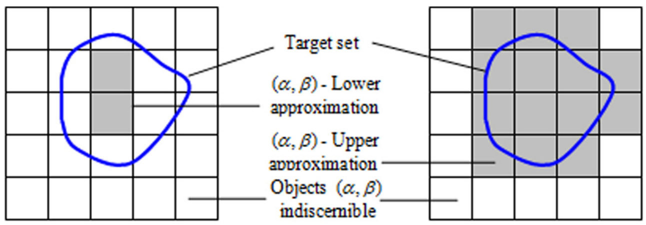

Fig. 3. Lower and upper approximation of rough set on intuitionistic fuzzy approximation.

$(\alpha,\beta) \in J$, where $J = \{(\alpha,\beta) : \alpha,\beta \in [0,1]\}$ and $0 \le (\alpha+\beta) \le 1$, the (α,β)-cut of R is given as:

$$R_{\alpha,\beta} = \{(q_i,q_j) \in (Q \times Q) : \mu_R(q_i,q_j) \ge \alpha, \nu_R(q_i,q_j) \le \beta\} \tag{18}$$

Let R be an intuitionistic proximity relation on Q. Two elements q_i and q_j are (α,β)-similar with respect to R if $(q_i,q_j) \in R_{\alpha,\beta}$ and it is written as $q_i R_{\alpha,\beta} q_j$. Two elements q_i and q_j are (α,β)-identical with respect to R for $(\alpha,\beta) \in J$, written as $q_i R(\alpha,\beta) q_j$ if and only if $q_i R_{\alpha,\beta} q_j$ or there exists a sequence of elements $q_1, q_2, q_3, \ldots, q_n$ in Q such that $q_i R_{\alpha,\beta} q_1, q_1 R_{\alpha,\beta} q_2, q_2 R_{\alpha,\beta} q_3, \ldots, q_n R_{\alpha,\beta} q_j$. In the last case, q_i is transitively (α,β)-similar to q_j with respect to R. It is also easy to see that for any $(\alpha,\beta) \in J$, $R(\alpha,\beta)$ is an equivalence relation on Q. Let $R^*_{\alpha,\beta}$ be the set of equivalence classes generated by the intuitionistic fuzzy proximity relation $R(\alpha,\beta)$ for each fixed $(\alpha,\beta) \in J$. The pair (Q,R) is an intuitionistic fuzzy approximation space. Let $X \subseteq Q$ be the target set. The lower and upper approximation of RSIFAS in the generalized approximation space $(Q, R(\alpha,\beta))$ is defined in Eqs. (19) and (20) respectively.

$$\underline{R}_{\alpha,\beta} X = \cup\{Y \in R^*_{\alpha,\beta} : Y \subseteq X\} \tag{19}$$

$$\overline{R}_{\alpha,\beta} X = \cup\{Y \in R^*_{\alpha,\beta} : Y \cap X \neq \phi\} \tag{20}$$

Equivalence classes obtained through this way are (α,β) equivalence classes. Objects belonging to such class are (α,β)-identical. On considering $\beta = 0$, RSIFAS reduces to RSFAS. Similarly, on considering $\alpha = 1$ and $\beta = 0$, RSIFAS reduces to rough set. The target set X is said to be (α,β)-rough if $\underline{R}_{\alpha,\beta} X \neq \overline{R}_{\alpha,\beta} X$. The (α,β)-boundary region of X is given as $BL_{R_{\alpha,\beta}}(X) = \overline{R}_{\alpha,\beta} X - \underline{R}_{\alpha,\beta} X$. The objects belonging to lower approximation are (α,β)-certain whereas objects belonging to $BL_{R_{\alpha,\beta}}(X)$ are uncertain. Fig. 3 depicts the (α,β)-lower and (α,β)-upper approximation of RSIFAS.

It is also verified that RSIFAS is a better model than RSFAS (Acharjya, 2009). The topological properties of RSIFAS and its characterization over basic set-theoretic operations, such as union and intersection on types of RSIFAS, are also discussed (Acharjya and Tripathy, 2009). These are useful while studying relational database systems.

3.8. Dominance based rough set

Rough set fails to analyze data containing preference order. It may lead to loss of information. On introducing preference order, the concept of dominance-based rough set (DRS) was introduced (Greco et al., 2000b,a) to overcome the limitations of a rough set. Given a decision system, there is a criterion, at least among condition attributes. Additionally, it is also observed that specific attributes like color, country may not be ordered. Therefore, criteria attributes are separated into ordered decision classes based on decision attribute. Also, conditional attributes are correlated semantically with ordered decision attributes employing dominance relation. In DRS, the equivalence relation of the rough set is replaced with dominance relation.

Dominance based rough set is defined on the notion of dominance principle to retrieve knowledge from a decision system. In such cases, based on decision class (P_d), the classification is carried out. Thus, the decision (P_d) divides the universe Q into finite number of classes, say $CL = \{CL_i : i \in Z\}; Z = \{1,2,3,\ldots,m\}$. In addition to this, these classes are ordered. It implies that, if $r, s \in Z$ and $r > s$, then the objects of class Cl_r are preferred then the objects of class Cl_s. The upward and downward unions of every element Cl_i of CL is given as $Cl_i^{\geq}$ and $Cl_i^{\leq}$ respectively. It is given as $Cl_i^{\geq} = \cup_{j \ge i} Cl_j$; and $Cl_i^{\leq} = \cup_{j \le i} Cl_j$.

Let $B \subseteq P_c$, objects certainly belongs to $Cl_i^{\geq}$ and $Cl_i^{\leq}$ are in their lower approximations $\underline{B}(Cl_i^{\geq})$ and $\underline{B}(Cl_i^{\leq})$ respectively. Similarly, objects possibly belong to $Cl_i^{\geq}$ and $Cl_i^{\leq}$ are in their upper approximations $\overline{B}(Cl_i^{\geq})$ and $\overline{B}(Cl_i^{\leq})$ respectively. The lower and upper approximation operators are defined as below.

$$\underline{B}(Cl_i^{\geq}) = \{q \in Q : D_B^+ \subseteq Cl_i^{\geq}\} \tag{21}$$

$$\underline{B}(Cl_i^{\leq}) = \{q \in Q : D_B^- \subseteq Cl_i^{\leq}\} \tag{22}$$

$$\overline{B}(Cl_i^{\geq}) = \cup_{q \in Cl_i^{\geq}} D_B^+(x) \tag{23}$$

$$\overline{B}(Cl_i^{\leq}) = \cup_{q \in Cl_i^{\leq}} D_B^-(x) \tag{24}$$

The boundary line objects of upward and downward union, $Cl_i^{\geq}$ and $Cl_i^{\geq}$ that contains ambiguous elements are defined as $BL_B(Cl_i^{\geq}) = \overline{B}(Cl_i^{\geq}) - \underline{B}(Cl_i^{\geq})$ and $BL_B(Cl_i^{\leq}) = \overline{B}(Cl_i^{\leq}) - \underline{B}(Cl_i^{\leq})$.

3.9. Covering based rough set

Rough sets introduced by Pawlak is a way to capture impreciseness. Imprecision in this notion is expressed by a boundary region. The boundary is due to lower and upper approximations of a set (Bonikowski et al., 1998; Du et al., 2011). The lower and upper approximation is defined with the help of equivalence relations. The equivalence relation generates partitions. A partition of the universal set Q is a set of sets $\{Q_1, Q_2, Q_3, \cdots, Q_k\}$ such that $Q = \cup Q_i$, $i = 1,2,\ldots,k$ and $(Q_i \cap Q_j) = \phi$ for $i \neq j$. But, in many applications $(Q_i \cap Q_j) \neq \phi$ and it leads to a covering instead of a partition. For example, consider a set of 10 institutions, $Q = \{q_1, q_2, q_3, \ldots, q_{10}\}$, to be judged on the parameters intellectual capital, infrastructure, placement, and recruiter satisfaction. Let the attribute values of intellectual capital are $\{high, average, low\}$. Let us assume a committee of four experts have to evaluate these institutions based on the parameters. Therefore, it is obvious that their decision in the same parameter may not be same as one another. For example, let us assume the decision of the committee members for intellectual capital as below:

$$\begin{array}{llll}
Expert_1: & High = \{q_1, q_4, q_6, q_8\} & Average = \{q_2, q_7\} & Low = \{q_3, q_5, q_9, q_{10}\} \\
Expert_2: & High = \{q_1, q_2, q_4, q_7, q_8\} & Average = \{q_3, q_6\} & Low = \{q_5, q_9, q_{10}\} \\
Expert_3: & High = \{q_1, q_3, q_4\} & Average = \{q_7\} & Low = \{q_2, q_5, q_6, q_8, q_9, q_{10}\} \\
Expert_4: & High = \{q_1, q_4, q_7\} & Average = \{q_8\} & Low = \{q_2, q_3, q_5, q_6, q_9, q_{10}\}
\end{array}$$

Since the decision of each member in the committee is of equal importance, we have to combine these decisions in order to avoid loss of information. Therefore, we should form the union of these decisions given by every committee member. Therefore, the classification obtained for parameter intellectual capital is $High = \{q_1, q_2, q_3, q_4, q_6, q_7, q_8\}$; $Average = \{q_2, q_3, q_6, q_7, q_8\}$ and $Low = \{q_2, q_3, q_5, q_6, q_8, q_9, q_{10}\}$. It is a cover instead of a partition. Such an argument may hold for all attributes.

Using covers instead of partitions, covering based rough sets (CBRS) has been introduced (Zakowski, 1983). According to this, there is only one way to define the lower approximation, whereas four definitions have been proposed for the upper approximation. Accordingly, four different types of covering based rough sets have been defined (Zhu and Wang, 2006; Zhu, 2006; Zhu and Wang, 2012, 2007). Before four types of covering rough sets are addressed, the notions and concepts related to it are presented.

Let Q be an universe and C be a cover of Q. We call (Q,C), the covering approximation space and the covering C is called the family of approximation sets. Let $q \in Q$. The minimal description of the object q is given as $Md(q) = \{K \in C : q \in K \wedge \forall T \in C (q \in T \wedge T \subseteq K \Rightarrow K = T)\}$.

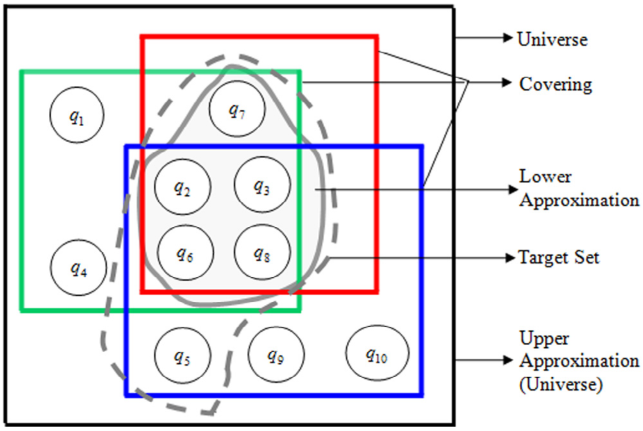

Fig. 4. Covering based rough set of type 2.

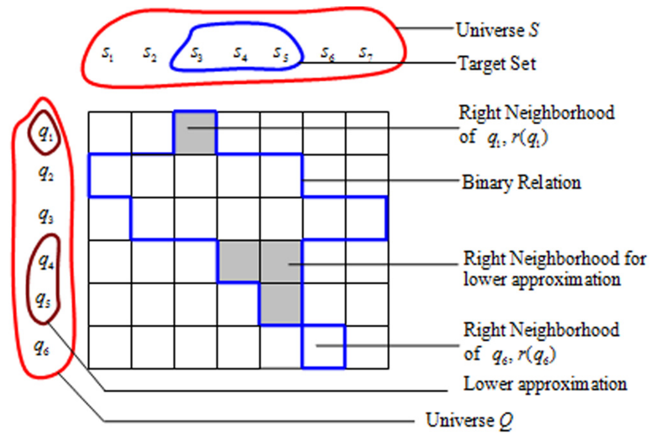

Fig. 5. Lower approximation of rough set on two universal sets.

Consider a target set $X \subseteq Q$. Let $\underline{C_i}$, $\overline{C_i}$, $i = 1,2,3,4$ be the lower and upper approximation of four different covering based rough sets respectively. These are defined as below.

$$\begin{aligned} \underline{C_1}(X) &= \cup\{K \in C : K \subseteq X\} \\ \overline{\overline{C_1}}(X) &= \underline{C_1}(X) \cup (\cup\{Md(q) : q \in (X - \underline{C_1}(X))\}) \end{aligned} \tag{25}$$

$$\begin{aligned} \underline{C_2}(X) &= \cup\{K \in C : K \subseteq X\} \\ \overline{\overline{C_2}}(X) &= \cup\{K \in C : K \cap X \neq \phi\} \end{aligned} \tag{26}$$

$$\begin{aligned} \underline{C_3}(X) &= \cup\{K \in C : K \subseteq X\} \\ \overline{\overline{C_3}}(X) &= \cup\{Md(q) : q \in X\} \end{aligned} \tag{27}$$

$$\begin{aligned} \underline{C_4}(X) &= \cup\{K \in C : K \subseteq X\} \\ \overline{\overline{C_4}}(X) &= \underline{C_4}(X) \cup \{K \in C : K \cap (X - \underline{C_4}(X)) \neq \phi\} \end{aligned} \tag{28}$$

From the definition it is clear that lower approximation of all different types of covering based rough sets are equal whereas upper approximations are different. The target set is said to be exact of $\underline{C_i}(X) = \overline{C_i}(X)$. The target set X is said to ith type covering based rough set if $\underline{C_i}(X) \neq \overline{C_i}(X)$ for $i = 1,2,3,4$ (Zhu, 2009; Yao and Yao, 2012). Let us assume the case discussed above. Let us consider the target set $X = \{q_2, q_3, q_5, q_6, q_7, q_8\}$. The lower approximation is given as $\underline{C_2}(X) = \{q_2, q_3, q_6, q_7, q_8\}$ whereas the upper approximation of second type of covering based rough set is the universe Q, i.e., $\overline{C_2}(X) = Q$. Fig. 4 depicts second type of covering based rough set. Similarly, computation for other types of covering based rough set can be carried out.

Further, more topological characterization on these types of covering based rough sets is also discussed (Zhu, 2007b). These four types of covering based rough sets are further extended to eight other types of covering based rough sets (Safari and Hooshmandasl, 2016).

3.10. Rough set on two universal sets

Rough set (Pawlak, 1982, 1981) is based upon the approximation of sets by a pair of lower and upper approximations. These approximation operators are indiscernibility relations. The restrictive condition of indiscernibility relation limits the application of the rough set model. Therefore, the rough set is generalized in many directions. For instance, indiscernibility relation is generalized to binary relations (Pawlak and Skowron, 2007b; Kondo, 2006; Zhu, 2007a), neighborhood systems, coverings (Zhu and Wang, 2007), Boolean algebras (Pawlak and Skowron, 2007a; Liu, 2005), fuzzy lattices (Liu, 2008), completely distributive lattices (Degang et al., 2006). Further, the rough set is generalized to rough set on two universal sets (RSTUS), and many of its algebraic properties are studied (Liu, 2010). We state in this section, the fundamental notions of the rough set on two universal sets.

Let Q and S be two universal sets and $R \subseteq (Q \times S)$ be a binary relation. The relational system (Q, S, R) is an approximation space. Let $q \in Q$ be an element. The right neighborhood of q in Q, $r(q)$ is defined as $r(q) = \cup\{s \in S : (q, s) \in R\}$. Similarly for an element $s \in S$, the left neighborhood of s in S, $l(s)$ is defined as $l(s) = \cup\{q \in Q : (q, s) \in R\}$.

Any two elements $q_1, q_2 \in Q$ are equivalent if $r(q_1) = r(q_2)$. Therefore, $(q_1, q_2) \in E_Q$ if and only if $r(q_1) = r(q_2)$, where E_Q denote the equivalence relation on Q. Therefore, E_Q partitions the universal set Q into disjoint subsets. Similarly, any two elements $s_1, s_2 \in S$ are equivalent if $l(s_1) = l(s_2)$. Thus, $(s_1, s_2) \in E_S$ if and only if $l(s_1) = l(s_2)$, where E_S denote the equivalence relation on S. It partitions the universal set S into disjoint subsets. Therefore, for approximation space (Q, S, R); $E_S \circ R = R = R \circ E_Q$, where $E_S \circ R$ is the composition of R and E_S. For any $Y \subseteq S$ and the binary relation R, the R-lower approximation $(\underline{R}Y)$ and R-upper approximations $(\overline{R}Y)$ of Y are defined as:

$$\underline{R}Y = \cup\{q \in Q : r(q) \subseteq Y\} \tag{29}$$

$$\overline{R}Y = \cup\{q \in Q : r(q) \cap Y \neq \phi\} \tag{30}$$

The R-boundary of Y is defined as $BL_R(Y) = \overline{R}Y - \underline{R}Y$, where R is the binary relation. The pair $(\underline{R}Y, \overline{R}Y)$ is called as the rough set of $Y \in S$ if $\underline{R}Y \neq \overline{R}Y$. Figs. 5 and 6 provides an overview of lower and upper approximation of rough set on two universal sets. Further approximation of classification and measure of uncertainty (Tripathy and Acharjya, 2012), topological characterization, rough equality of sets on two universal sets (Tripathy et al., 2011b; Acharjya and Tripathy, 2013), and inclusion of sets are expressed in terms of binary relation. Results obtained are important for their application in the design of knowledge bases. Additionally, general characterization of classification to predict all possible combinations of types of elements for a classification of 2 and 3 numbers of elements is established (Das et al., 2015a; Acharjya, 2014).

3.11. Fuzzy rough set on two universal sets

Rough sets of Pawlak, as discussed earlier, depend on equivalence relations, and it is restrictive in many applications. Further, an equivalence relation is generalized to binary relation, and the concept of the rough set on two universal sets is introduced. Further, a fuzzy relation is more applicable in real-life applications, and the concept of a fuzzy rough set on two universal sets is introduced (Liu, 2010). It generalizes the fuzzy rough set of Dubois and Prade (Dubois and Prade, 1990) to a fuzzy rough set on two universal sets (FRSTUS). Now, we state the definitions and notions of fuzzy rough sets on two universal sets.

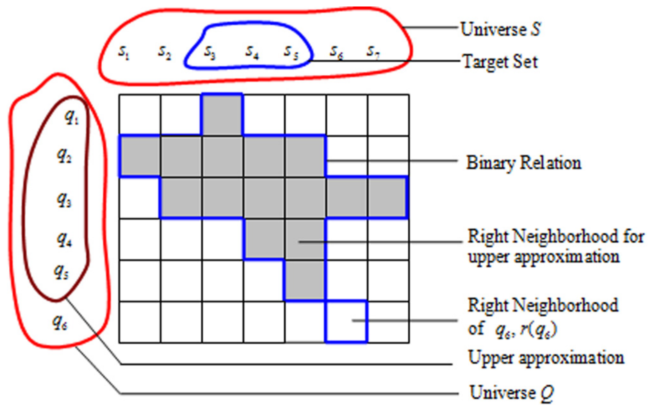

Fig. 6. Upper approximation of rough set on two universal sets.

Let Q be an universe and q is an object of Q. A fuzzy set X of Q is defined as a collection of ordered pairs $(q, \mu_X(q))$, where $\mu_X(q) : Q \to [0,1]$ is a mapping known as the membership function of X. The family of all fuzzy sets in Q is denoted as $F(Q)$. Let Q and S be two non empty universal sets. Let R_F be a fuzzy binary relation from $Q \to S$. Therefore, (Q, S, R_F) is called a fuzzy approximation space. For any $Y \in F(S)$ and the fuzzy binary relation R_F, we associate two subsets $\underline{R}_F Y$ and $\overline{R}_F Y$ called the R_F-lower and R_F-upper approximations of Y respectively. A fuzzy rough set on two universal set (Liu, 2010) is a pair $(\underline{R}_F Y, \overline{R}_F Y)$ of fuzzy sets on Q such that for every $q \in Q$:

$$(\underline{R}_F Y)q = \wedge_{s\in S}((1 - \mu_{R_F}(q,s)) \vee Y(s)) \tag{31}$$

$$(\overline{R}_F Y)q = \vee_{s\in S}(\mu_{R_F}(q,s) \wedge Y(s)) \tag{32}$$

G. Liu (Liu, 2010) discusses fundamental algebraic properties of a fuzzy rough set on two universal sets. Further topological characterization of the fuzzy rough set on two universal sets and its application to multicriteria decision making is being carried out (Acharjya, 2014).

3.12. Intuitionistic fuzzy rough set on two universal sets

Guilong Liu (Liu, 2010) introduced the fuzzy rough set on two universal sets. We do not consider the non-membership in a fuzzy set; instead, we assume membership of all elements exists. In many real-life problems, it is not true because of hesitation. If $\mu(q)$ be the membership of an element q, then the non-membership of q is computed using $(1 - \mu(q))$ in the fuzzy set with the assumption that full part of the membership is determinism and indeterministic part is zero. It is not always true, and hence the intuitionistic fuzzy set is better. It indicates that the intuitionistic fuzzy set model is a generalized model over the fuzzy set model. Therefore, the intuitionistic fuzzy rough set on two universal sets (IFRSTUS) is a better model than a fuzzy rough set on two universal sets (Acharjya and Tripathy, 2012). Following, we briefly state the definitions, notations, and results of the intuitionistic fuzzy rough set on two universal sets. We use μ for membership and ν for non-membership to define basic notions of the intuitionistic fuzzy rough set on two universal sets.

Let Q be an universe and $q \in Q$. An intuitionistic fuzzy set X of Q is defined as $\langle q, \mu_X(q), \nu_X(q)\rangle$, where $\mu_X : Q \to [0,1]$ and $\nu_X : Q \to [0,1]$ represents the membership and non membership respectively of the element $q \in Q$ to the set X. For every $q \in Q$, $0 \le \mu_X(q) + \nu_X(q) \le 1$. The amount $\pi_X(q)$ is called the hesitation part such that $\pi_X(q) = 1 - (\mu_X(q) + \nu_X(q))$, which may cater either membership or nonmembership or the both. For simplicity, (μ_X, ν_X) is used to denote the intuitionistic fuzzy set X. The family of all intuitionistic fuzzy subsets of Q is denoted as $IF(Q)$.

Let Q and S be two non empty universal sets. An intuitionistic fuzzy relation R_{IF} from $Q \to S$ is an intuitionistic fuzzy set of $(Q \times S)$ characterized by the membership μ_R and non-membership ν_R, where

$R_{IF} = \{\langle (q,s), \mu_{R_{IF}}(q,s), \nu_{R_{IF}}(q,s)\rangle | q \in Q, s \in S\}$

with $0 \le \mu_{R_{IF}}(q,s) + \nu_{R_{IF}}(q,s) \le 1$ for every $(q,s) \in (Q \times S)$. If for $q \in Q$, $\mu_{R_{IF}}(q,s) = 0$ and $\nu_{R_{IF}}(q,s) = 1$ for all $s \in S$, we call q is a solitary element with respect to R_{IF}. The set of all solitary elements with respect to the relation R_{IF} is called solitary set (Acharjya and Tripathy, 2012).

Let Q and S be two non empty universes and R_{IF} be an intuitionistic fuzzy relation from $Q \to S$. Therefore, (Q, S, R_{IF}) is called an intuitionistic fuzzy approximation space. For $Y \in IF(S)$, an intuitionistic fuzzy rough set on two universal sets is a pair $(\underline{R}_{IF}Y, \overline{R}_{IF}Y)$ of intuitionistic fuzzy set on Q such that for every $q \in Q$,

$$\underline{R}_{IF}Y = \{\langle q, \mu_{\underline{R}_{IF}(Y)}(q), \nu_{\underline{R}_{IF}(Y)}(q)\rangle | q \in Q\} \tag{33}$$

$$\overline{R}_{IF}Y = \{\langle q, \mu_{\overline{R}_{IF}(Y)}(q), \nu_{\overline{R}_{IF}(Y)}(q)\rangle | q \in Q\} \tag{34}$$

where

$$\begin{aligned}
\mu_{\underline{R}_{IF}(Y)}(q) &= \wedge_{s\in S}[\nu_{R_{IF}}(q,s) \vee \mu_Y(s)] \\
\nu_{\underline{R}_{IF}(Y)}(q) &= \vee_{s\in S}[\mu_{R_{IF}}(q,s) \wedge \nu_Y(s)] \\
\mu_{\overline{R}_{IF}(Y)}(q) &= \vee_{s\in S}[\mu_{R_{IF}}(q,s) \wedge \mu_Y(s)] \\
\nu_{\overline{R}_{IF}(Y)}(q) &= \wedge_{s\in S}[\nu_{R_{IF}}(q,s) \vee \nu_Y(s)]
\end{aligned}$$

The pair $(\underline{R}_{IF}Y, \overline{R}_{IF}Y)$ is known as intuitionistic fuzzy rough set on two universal sets of Y concerning (Q, S, R_{IF}), where $\underline{R}_{IF}Y$, $\overline{R}_{IF}Y : IF(Q) \to IF(S)$ are referred as lower and upper intuitionistic fuzzy rough approximation operators on two universal sets. Algebraic properties of the intuitionistic fuzzy rough set on two universal sets following rough set theory (Acharjya, 2014) and its application to multicriteria decision making (Das et al., 2015b) is discussed in the literature.

3.13. Multi granular rough set

Considering granular computing, a target set is represented using a lower and upper approximation in single granulation. It means that the concept is developed using one equivalence relation. Nevertheless, in many applications, one equivalence relation may not be suitable to analyze a problem. Therefore, more than one equivalence relation is employed to handle such applications. It leads to the concept of multi-granulation, and the notion of a multi granular rough set (MGRS) is introduced (Qian and Liang, 2006; Qian et al., 2007). Now we define the notion and concepts of MGRS briefly.

Let $Q = \{q_1, q_2, \ldots, q_n\}$ be the objects of the universal set, and $\mathfrak{R}$ be a family of equivalence relations defined on Q. Let $R_1 \subseteq (Q \times Q)$ and $R_2 \subseteq (Q \times Q)$ be two equivalence relations defined on Q. Let us consider a target set $X \subseteq Q$ that is approximated by two equivalence relations R_1 and R_2. The lower and upper approximation of MGRS is defined as in Eqs. (35) and (36) respectively, where X^c is the complement of X.

$$\underline{(R_1 + R_2)}X = \cup\{q \in Q : [q]_{R_1} \subseteq X \ or \ [q]_{R_2} \subseteq X\} \tag{35}$$

$$\overline{(R_1 + R_2)}X = \{\underline{(R_1 + R_2)}X^c\}^c \tag{36}$$

The boundary line objects is defined as $BL_{R_1+R_2}(X)$, where $BL_{R_1+R_2}(X) = \overline{(R_1 + R_2)}X - \underline{(R_1 + R_2)}X$. It indicates that, the target set X is said to be $(R_1 + R_2)$ certain if $BL_{R_1+R_2}(X) = \phi$. Equivalently, it can be written as $\overline{(R_1 + R_2)}X = \underline{(R_1 + R_2)}X$. Similarly, the target set X is said to be $(R_1 + R_2)$ MGRS if $BL_{R_1+R_2}(X) \neq \phi$. Likewise, it can be written as $\overline{(R_1 + R_2)}X \neq \underline{(R_1 + R_2)}X$. The above Eqs. (35) and (36) are defined over two equivalence relations. A graphical view of Pawlak's rough set and multigranular rough set is depicted in Fig. 7. The Fig. 7 is restricted to two different equivalence relations. However, it can be generalized to any finite number of equivalence relations. From Fig. 7 it is clear that, MGRS covers more number of objects as compared to traditional rough set.

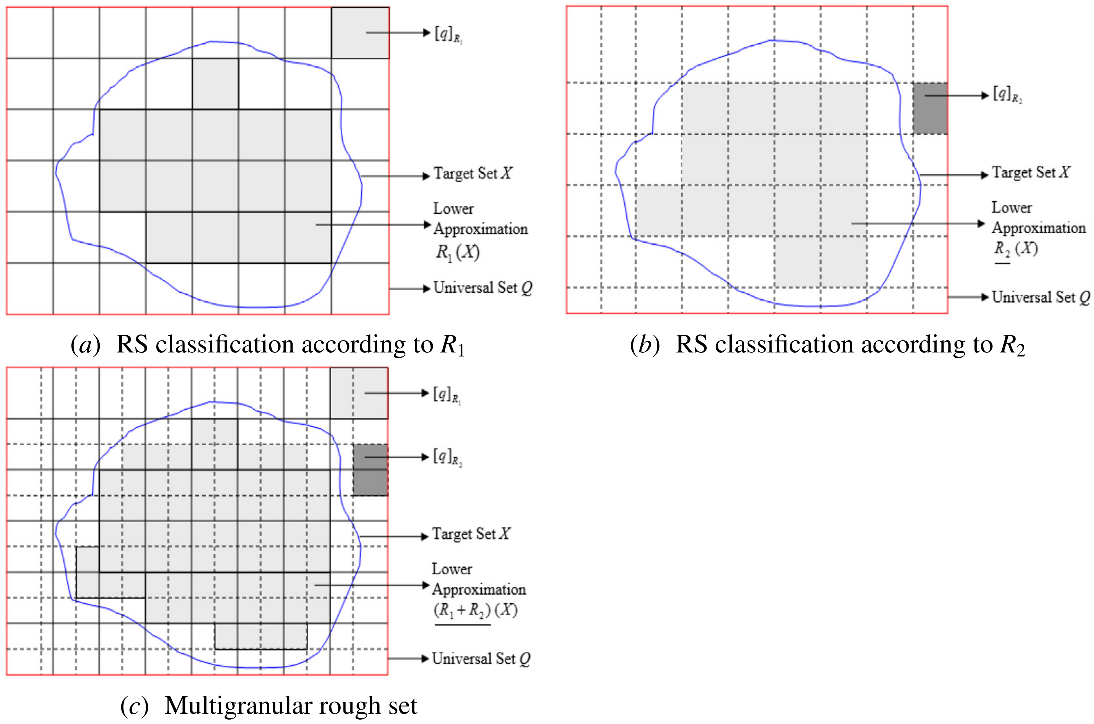

Fig. 7. Lower approximation representation of MGRS.

Further, the topological properties (Tripathy and Nagaraju, 2012), approximate equivalences (Tripathy and Mitra, 2013), approximation of classifications (Raghavan and Tripathy, 2013), approximate equalities of MGRS are also studied in the literature. Similarly, MGRS has been extended to many extents, such as covering based MGRS (Liu et al., 2014), MGRS based on tolerance relation (Xu et al., 2013), MGRS on two universal sets (Geetha et al., 2014, 2013), and some more. Mentioning all those extensions is beyond the scope of this research work.

4. Hybridized rough computing

The rough set and its variations are hybridized with many other concepts, such as neural network, genetic algorithm, support vector machine, bioinspired computing, and many more to provide promising results. These hybridized concepts are primarily applied to various real-life problems. These applications are mainly dealing with classification, feature selection, knowledge discovery, decision rule optimization, and much more. This research work mainly focuses on basic concepts of hybridization. In the subsequent subsections, it highlights those hybridizations which are widely used in computer science and real-life applications.

4.1. Hybridization of rough set and neural network

The rough set introduced by Pawlak (1982) has specific limitations while considering generative aspects of rules. It generates both precise and approximate rules. It is due to the presence of boundary regions. As a result, the accuracy of the developed model is reduced in some real-life problems. To overcome such limitation, Anitha (Ahn et al., 2000; Anitha and Acharjya, 2015) hybridized rough set with the neural network (RSNN) to achieve better results, which a standalone technique cannot produce. The model developed is a two-phase system in which the rough set is used for both horizontal and vertical reduction. In the second phase, the neural network predicts the decision value of the unseen associations of attribute values. In this model, the back-propagation neural network is used to train the model. Simultaneously, the sigmoid function is used as an activation function in the network.

The prime objective of these models is to handle the non-deterministic behavior of the rough set. It is also observed that sometimes the rough set cannot predict newly introduced unseen associations of attribute values. It is because of non-deterministic rules due to rough set data analysis while generating rules. This model cannot be applied directly if the input is a real-valued information system. In such cases, the model employs some discretization procedure to reduce the real-valued information system to a qualitative information system. To overcome this, the rough set on fuzzy approximation space is hybridized with neural networks and studied over the agriculture information system (Anitha and Acharjya, 2018). Similarly, for identifying salient features present in unsupervised data, a granular neural network is integrated with a fuzzy rough set (Ganivada et al., 2013). Further, a fuzzy rough set on two universal sets is integrated with a radial basis function neural network for knowledge acquisition. It is studied for finding customer choice of supermarkets in a city (Anitha and Acharjya, 2016). A comparative study of statistical methods and hybridized rough computing with neural network methods are also available. The comparative study is carried out over business failure prediction (Acharjya and Anitha, 2017).

4.2. Hybridization of rough set and genetic algorithm

The rough set is widely used to handle uncertainties in an information system. It computes reducts and generates decision rules. However, the rough set generates too many rules that include both precise and approximate rules. In general, the rough set is just imposed as a technique to identify superfluous attributes. However, in none of the models, the approximate rules are not optimized to provide better results. To overcome this limitation, the hybridization of the rough set and genetic algorithm is carried out. The proposed model offers a sound methodology, mines rules efficiently, and refine the approximate rules to obtain better accuracy. It integrates two techniques, such as the rough set and binary-coded genetic algorithm (Rathi and Acharjya, 2018b). In this model, the approximate rules with accuracy more than the threshold value are passed to the next phase for processing with the binary-coded genetic algorithm to refine rules. Based on the approximate rules, the initial population concerning the chromosome is generated. The chromosomes are coded for the approximate rules generated from the rough set.

The rough set with a binary-coded genetic algorithm model optimizes the approximate rules generated due to rough set. However, the major limitation is that it requires more search space when an information system contains many parameters. To overcome this limitation, the binary-coded genetic algorithm is replaced with a real-coded genetic algorithm. Further, another hybridization of the rough set and real-coded genetic algorithm is developed. The proposed model integrates the rough set, real-coded genetic algorithm, regression analysis, and the KNN algorithm (Rathi and Acharjya, 2018a). Besides, rough set in integration with genetic algorithm is studied over feature reduction (Jing, 2014; Das et al., 2018), rule mining system (Khoo and Zhai, 2001), and prediction (Yu et al., 2009).

4.3. Hybridization of rough set and particle swarm optimization

At present, data are growing at a high rate. Finding optimal features from the information system is a critical task. To this, many algorithms are proposed. One such algorithm is the hybridization of particle swarm optimization (PSO) and rough set. The PSO is often integrated with a rough set for optimal feature selection (Wang et al., 2007). Similarly, the rough set is also integrated with an intelligent dynamic swarm for feature selection. It overcomes the premature convergence of PSOp (Bae et al., 2010). In the other direction, minimal reduct of the rough set is computed using PSO. It is designed to overcome the complex operators of genetic algorithm (Wang et al., 2005). In another concept, PSO is used for obtaining a reduct on analyzing the positive region of rough set (Yue et al., 2007).

Additionally, the cost attribute reduct problem is discussed using an integrated approach that integrates PSO and positive regions of the rough set. The evaluation matrices are designed using standard uniform, normal, and Pareto distribution. The main focus is to compute minimal test cost reduct (Dai et al., 2016). Correspondingly, integrating decision-theoretic rough set and PSO, a heuristic technique is also proposed for feature reduction (Jia et al., 2014).

4.4. Hybridization of the rough set with bioinspired computing

Hybridization of the rough set with bioinspired computing is seen in many healthcare applications. Mainly the hybridization is used for feature selection. Feature selection is an NP-hard problem. Binary ant lion optimization is hybridized with rough set and approximate entropy (Mafarja and Mirjalili, 2019) to solve this NP-hard problem. Besides, the rough set is also hybridized with an artificial bee colony algorithm. It is further analyzed over the healthcare domain for feature selection (Suguna and Thanushkodi, 2010) and leak detection analysis (Mandal et al., 2012). Similarly, the rough set is widely integrated with ant colony optimization for computation of reduct (Jensen and Shen, 2003) and feature selection (Chen et al., 2010b; Ke et al., 2008). Furthermore, the rough set is integrated with the firefly algorithm for the extraction of main features, and it is analyzed over brain tumor classification (Jothi and Hannah, 2016). Furthermore, attribute reduction is carried out using a rough set and hybrid artificial bee colony algorithm (Chebrolu and Sanjeevi, 2017). Similarly, rough set, artificial bee colony, and neural network are integrated and studied for multiple classifications.

In some applications, the rough set is hybridized with bioinspired computing techniques in two phases. Bioinspired computing is applied in the first phase to get the chief factors in such hybridizations, whereas a rough set is applied in the second phase for decision rule generation. It helps in developing expert systems and early diagnosis of diseases (Kauser and Acharjya, 2020). A review of swarm intelligence algorithms and the rough set is found in the literature (Acharjya and Kauser, 2015).

4.5. Hybridization of rough set and support vector machine

Like feature selection, classification is also of prime importance in machine learning. The rough set is widely used for classification problems. In some applications, rough set and support vector machine (SVM) are integrated to provide better results. In most of the applications, a rough set is applied for feature selection, whereas SVM is used for classification (Chen et al., 2011). In some cases, a rough set is used for dimensionality reduction, whereas SVM is used for classification (Chen et al., 2009). The main advantage is that the rough set eliminates the necessity of exact classification while dealing with noisy data (Lingras and Butz, 2007). Besides, for pattern classification and attribute reduction, a rough neighborhood set is integrated with the support vector machine is studied. It is studied over the mechanical fault diagnosis problem (Li et al., 2012). This hybridization is widely studied over the intrusion detection system, anomaly detection, financial distress, and fault diagnosis. Additionally, FRS is also hybridized with SVM. The main aim of this hybridization is to provide membership to every training sample in constraints. Similarly, fuzzy SVM is introduced for dealing with outliers (Chen et al., 2010a; He and Wu, 2011).

Furthermore, rough computing is hybridized with many other concepts such as formal concept analysis, structural equation modeling, partial least square, and much more. The hybridization and core concepts are applied recently to cultural heritage, wearable computing, ambient intelligence, image processing, etc.

4.6. Hybridization of rough computing and formal concept analysis

Rough computing and formal concept analysis (FCA) aims at different perceptions. Rough computing aims at prediction, whereas FCA aims at description. Thus, hybridization refers to a better model. It is mainly used for identifying the principal attribute values that affect the decisions. It, in turn, help the decision-makers to take appropriate decisions. In the initial phase of the hybridization, rough computing is employed to generate decision rules. These decision rules are further processed with FCA in the final stage to identify the principal attribute values affecting the decision. So far rough set is hybridized with formal concept analysis and studied over a variety of scientific problems (Lai and Zhang, 2009; Kang et al., 2013). The hybridization is studied over heart disease diagnosis (Tripathy et al., 2011a), intrusion detection system for identifying phishing attacks (Ahmed et al., 2017), and personal investment portfolios (Shyng et al., 2010). Similarly, RSFAS is hybridized with FCA and studied over attribute selection in marketing (Acharjya and Das, 2017).

5. Software libraries of rough set theory

Software libraries available for the rough set is very limited. Though many variations are developed in perception with the rough set, software development for different variations is very restricted. At this moment, individuals and institutions are using *R* software for knowledge discovery, data logic. The software *R* is open-source and freely available software. The software *R* is licensed under the terms of the GNU General Public License. Reduct System Inc mostly uses it in Canada for knowledge discovery and data logic. Similarly, the software Python is mainly used for feature selection. The software Python is developed under an OSI-approved open source license. Besides, the University of Texas, USA, has developed a rough set rule mining tool using learning from examples on rough sets (LERS) algorithm. It supports a large number of complex data. Besides, it has been widely adopted by NASA's space center. They use it mainly for expert systems.

Moreover, it is also explored by the Environmental Protection Agency, USA.

Similarly, Poznan University of Technology, Poland, has developed a rough set data analysis system for computing core and reduct. Likewise, Chongqing University of Posts and Telecommunications, China, has also developed a rough set intelligent data analysis system (RIDAS) for analyzing uncertain, imprecise, and vague information (Wang et al., 2002). Further, Infobright Community Edition (ICE) has developed an analytic database engine for handling approximate rule generation (Slzak et al., 2010). But, all these software developments are restricted to rough set data analysis. Though rough set has extended to FRS, PRS, DTRS, VPRS, RSFAS, RSIFAS, DRS, CBRS, RSTUS, FRSTUS, IFRSTUS, and MGRS, software for analyzing data analysis using these variations is rarely found. Researchers in general use, *R*, Python, and *C* programming language for their application development.

6. Rough computing applications

Rough computing applications mainly refers to feature reduction, knowledge discovery, decision rule generation, and intelligent algorithm development. The reduction of features in an information system is an NP-hard problem. The growth of rough computing and its hybridization with other algorithms has given a path for knowledge discovery and data mining. With the development of information entropy, nature-inspired algorithms, rough computing has attained numerous accomplishments while handling various information systems. At this moment in time, primarily, the research is carried out in three different directions, such as theoretical advancement, algorithm development, and applications. The theoretical advancement of a rough set is discussed in abstraction. In this section, the various applications of rough computing and its hybridizations are briefly presented.

1. Rough computing in knowledge representation
    - Knowledge discovery and uncertainty analysis (Dubois and Prade, 1987; Grzymala-Busse, 1988; Zhong, 2001; Wang et al., 2010).
    - Distributed knowledge system (Acharjya and Tripathy, 2008, 2009).
2. Rough computing and its hybridization in prediction and fault diagnosis
    - Evaluation of bankruptcy risk and prediction (Slowinski and Zopounidis, 1995; McKee, 2003; Chuang, 2013; Mckee, 2000).
    - Business failure prediction (Ahn et al., 2000; Xiao et al., 2012).
    - Fault diagnosis (Shen et al., 2000; Li et al., 2012; Tay and Shen, 2003; Wang and Li, 2004; Geng and Zhu, 2009).
    - Predictive data analysis (Acharjya and Anitha, 2017; Yeh et al., 2014).
    - Customer choice of supermarket (Anitha and Acharjya, 2016)
3. Rough computing and its hybridization in agriculture science
    - Agriculture decision system using rough set (Barbagallo et al., 2006).
    - Agriculture crop prediction using rough computing and neural network (Anitha and Acharjya, 2015, 2018).
    - Agriculture crop prediction using rough computing and genetic algorithm (Rathi and Acharjya, 2018b,a).
    - Agriculture sustainability (Demartini et al., 2015).
4. Rough computing and decision rule making
    - Pilgrimage attitude towards cultural heritage (Acharjya and Acharjya, 2020; Au and Law, 2000).
    - Multicriteria decision making (Das et al., 2015b; Li and Zhou, 2011; Xie et al., 2008; Sun et al., 2017).
    - Ambient intelligence and behavioral intention (Gnana et al., 2020; Acharjya and Natarajan, 2019).
    - Intrusion and detection system (Chen et al., 2009; Adetunmbi et al., 2008; Zhang et al., 2004; Cai et al., 2003; Ahmed and Acharjya, 2015, 2019).
5. Application of rough computing and its hybridizations in healthcare
    - Brain tumor image classification (Jothi and Hannah, 2016).
    - Heart disease diagnosis (Kauser and Acharjya, 2020; Tripathy et al., 2011a; Son et al., 2012).
    - Breast cancer diagnosis (Chen et al., 2011; Chowdhary and Acharjya, 2016; Chen et al., 2011).
    - Nursing informatics and information technology usage (Singh and Acharjya, 2020).
6. Rough computing and its hybridization in industrial applications
    - Intelligent industrial applications (Pawlak, 2000).
    - Industrial wastewater treatment (Chen et al., 2003).
    - Circuit board manufacturing (Tseng et al., 2004).
    - Remote monitoring and diagnosis of manufacturing (Hou et al., 2003).

Besides the applications listed here, several other applications are employed in which a rough set and its hybridizations are employed. To name a few, rough computing is also applied in power system, image processing, massive data processing, e-mail filtering, pattern recognition, and gene expression. Listing all applications of rough computing and its hybridization is a critical task, and it is beyond the scope of this research work. However, the applications that widely use rough computing and its hybridization is listed below.

6.1. Rough computing in knowledge representation

There are representation techniques that originated from human information processing theories are frames, rules, tagging, and semantic networks. Intelligent behavior is achieved due to knowledge representation, and reasoning is an area in artificial intelligence that is concerned with formally thinking and representing knowledge to facilitate inferencing from expertise. Rough computing helps in knowledge representation and is domain-independent. Besides, it has the expressivity of the representation scheme in terms of formal language.

Much research has been carried out in extracting knowledge from an information system using a rough set. Knowledge extraction, classification, feature selection, and prediction are widely explored using rough computing. It has been applied in many real-life applications. For example, performance evaluation and ranking of institutions (Acharjya and Ezhilarasi, 2011; Tripathy and Acharjya, 2010; Acharjya and Bhattacharjee, 2013), sustainability into supplier selection (Bai and Sarkis, 2010), concept evaluation in product development (Zhai et al., 2009), uncertainty measure and feature selection (Zhong et al., 2001; Liang et al., 2009; Liang and Shi, 2004), and medical diagnosis (Hassanien, 2007; Placzek, 2013; Chowdhary and Acharjya, 2016). Besides, there are several other applications also available in the literature.

6.2. Rough computing in marketing and financial forecasting

The primary application in marketing is a database marketing system. The rough set helps in analyzing customer databases to identify different customer groups and predict their behavior. Another such form is market basket analysis, in which it determines the pattern of a customer. Similarly, the rough set is also widely used in economic and financial forecasting. The primary and essential areas in which rough set theory is used mainly in economic and financial prediction are business failure prediction, database marketing, and financial investment (Ahn et al., 2000; McKee, 2003; Chuang, 2013).

Business failure prediction is a scientific field in which many academics and professionals are interested. Financial institutions like banks, credit centers, and clients need these predictions for evaluating firms in which they have an interest. Various methods, such as discriminant analysis, logic analysis, profit analysis, and recursive partitioning algorithms, have been applied to model such real-life problems. Database marketing is related to thinking and acting, in which it contains the application tools and methods in studies. The goals are the formation of industries surroundings, their structure, and internal organization that they follow to achieve success on a fluctuating consumer marketing. Technically, database marketing can be defined as a method analyzing customer data to look at patterns among existing preferences and use these patterns to target customers more. Financial investment analysis applications employ predictive modeling techniques, such as statistical regression and neural networks, to create and optimize portfolios and build trading systems. Building trading systems using rough set theory has been studied by several researchers (Xiao et al., 2012; Yeh et al., 2014).

Moreover, customer satisfaction analysis is another original concept in strategic management. In such cases, the customer databases are explored to identify a different group of customers and recognize their shopping patterns. It, in turn, help to predict the behavior of the customers. Such applications are called marked basket analysis. The customers' preference is generated as decision rules to measure the expected efficiency of the strategic inventions. As a result, services offered to customer satisfaction-oriented management, and the quality of the product is improved (Blaszczynski et al., 2007).

6.3. Rough computing in agriculture science

The backbone of India is agriculture, and the majority of people depend on agriculture. Information gathering in agriculture is essential before crop cultivation. The crop cultivation depends on several factors, such as micro-nutrients and macro-nutrients. The plant receives these nutrients from soil and water. Therefore, agriculture productivity depends on prior information about various intricate factors. The agriculture data is mostly inconsistent and contains uncertainties. The rough set and its hybridizations are used to identify suitable crops by analyzing inconsistencies and ambiguities present in the crop information system. The rough set theory helps in reducing data and does not require any prior knowledge for analysis. Therefore, the rough set theory can be used to acquire more knowledge in agriculture cultivation and identify the defect to improve the yield (Jianping, 2009; Demartini et al., 2015).

Furthermore, rough set and RSFAS is hybridized with neural network and studied on crop suitability identification on Vellore district of Tamil Nadu, India (Anitha and Acharjya, 2015, 2018). This hybridization's prime objective is to identify a suitable crop for cultivation based on macro and micro-nutrients. Similarly, in another context, a rough set is hybridized with a binary-coded genetic algorithm and a real coded genetic algorithm to study the crop's prediction to be cultivated based on micro and macro-nutrients. The study is carried out on the Tiruvannamalai district of Tamil Nadu, India (Rathi and Acharjya, 2018b,a).

6.4. Rough computing and healthcare

A sizeable worldwide amount of data is generated every moment from various sources. Healthcare consortium and nursing informatics are one among them. But, starving knowledge from information is a critical concern in recent years. It characterizes the necessity and justification of knowledge discovery. Besides, analysis of uncertainty and impreciseness present in the healthcare data is another primary concern in recent years. It leads to a prime area of healthcare research. Many techniques are developed to analyze dilemmas, impreciseness present in data such as computational intelligence, and intelligent computing. Rough computing plays a vital role in feature selection and rule extraction. It, in turn, help physicians to make appropriate decisions. Generally, medical datasets occupy an enormous amount of data about diseases, patients, and physicians. The cost of disease prediction can be approximately reduced, and the time taken for diagnosis can be reduced further, by applying the feature selection technique of rough set theory (Wang and Ma, 2009). Also, the rough set's rule generation process plays a significant role in the prediction and decision-making process in health care industries.

Rough set, along with formal concept analysis, is used in heart disease diagnosis for identifying the chief factors affecting the decisions (Tripathy et al., 2011a). Likewise, heart disease diagnosis using several hybridized rough computing is also found in the literature (Srinivas et al., 2014; Africa, 2016; Mitra et al., 2006). Furthermore, rough set is also used to diagnose hepatitis disease (Kaya and Uyar, 2013), brain tumor image classification (Chen et al., 2010a), breast cancer disease (Jothi and Hannah, 2016; Hassanien, 2003), kidney cancer (Saleem Durai et al., 2012) etc. Similarly, many other healthcare applications using rough set is also found in the literature.

6.5. Rough computing in behavioral intention and cultural heritage

Development builds upon understanding. Various persons work on multiple domains. Ever it is necessary to figure out the feelings of the persons on the development process's success. It categorized towards the behavioral intention of the person. Behavioral intention can be defined as a person's observed presumption. It is also stated as the subjective probability of a person's behavior. A simple behavioral intention maybe "I intend to use the smartphone" it can be pin down as to why, which, what, and when. The above statement can be voiced similarly by many persons, but the pin-down version differs from person to person. It shows the person's expectation around their behavior in the given scenarios. It is observed that a person's behavior contains uncertainties, and thus there is a need for intelligent techniques to process behavioral intention. A fuzzy rough set is recently used to study the investor's behavior towards investing in the gold exchange-traded fund (Acharjya and Natarajan, 2019). Similarly, in another context, a fuzzy rough set is employed to analyze customers' behavioral intention towards smartwatches in an ambient environment (Gladys and Acharjya, 2020). Many studies have not been carried out in behavior intention and ambient intelligence, applying rough computing.

Homogeneously, cultural heritage is another upcoming area of research in social and computer science. People are neglected towards their cultural heritage because of fast growth in information and communication technology and several other factors. As a result, the transformation of cultural heritage from generation to generation is getting deteriorated. It dramatically impacts on pilgrimage attitude towards cultural heritage. Besides, the expansion of heritage places improves the economic worth of any nation. Further, pilgrimage attraction is a significant concern, which in turn enhances the business opportunities. The other concepts associated with cultural heritage are the behavioral intention of pilgrimage, sightseeing expenditures, and travel research. The information system that generated from cultural heritage contains uncertainties. A rough set approach for studying pilgrimage attitude towards cultural heritage is found in the literature (Acharjya and Acharjya, 2020). Similarly, sightseeing expenditure in cultural heritage has been carried out using a rough set (Au and Law, 2000). Likewise, travel attribute analysis has been carried out using a rough set and neural network approach (Golmohammadi et al., 2011). But, less research is found in cultural heritage literature using rough sets and other intelligent techniques.

6.6. Rough computing and sentiment analysis

Sentiment analysis is the classification and interpretation of emotions inside text data using text analysis techniques. These emotions are associated with social media data analysis and thus natural language processing is a way to understand the language and to unveil the emotions associated with text data. Much work in sentiment analysis using rough computing and its hybridization is not carried out by researcher. A hybridized approach that integrates rough set and support vector machine is employed to recognize emotions from text. The proposed approach devise a decision table with chief attributions. It, in turn, is analyzed in accordance with relative reduction (Teng et al., 2007). In another context rough set is used in analyzing public tweets in twitter. The prime concept used in this work is to segregate the positive, negative and neutral sentiments (Das et al., 2014a). Likewise, online product review employing rough set rule induction is carried out. The prime aim in this work is to filter out the positive, negative, and neural emotions of an online product (Das et al., 2014b).

7. Conclusion and future extension

The rough computing research has been carried out by many researchers since its inception more than 35 years. It has remarkable achievements in many areas, like Knowledge representation, information retrieval, machine learning, knowledge discovery in database. It has been applied to many fields, like uncertain information analysis, distributed knowledge system, fault diagnosis, predictive analysis, agriculture, decision support system, agriculture sustainability, cultural heritage, behavioral intension, intrusion detection, image processing, healthcare system, nursing informatics, pattern recognition, industrial application, management and so on. In general, it has applied to all areas, and its application range is extensive. Nevertheless, it is observed that the abstractions developed from a classical rough set are rarely applied to various applications. Similarly, handling big data, working with parallel architecture, hybridized reduct, and decision rule generation algorithms are not addressed in the literature effectively. All these are challenging, and it can be concluded that rough computing has enormous growth shortly.

Rough set has been widely used in almost all areas of applications. But, the variations of rough set such as FRS, PRS, DTRS, VPRS, RSFAS, RSIFAS, DRS, CBRS, RSTUS, FRSTUS, IFRSTUS, and MGRS has limited real life applications. Therefore, all real life applications could be explored by employing these variations. Further, these applications could be compared with the existing applications. Likewise very limited number of hybridizations of different concepts with rough computing is found in the literature. Therefore, further research can be fostered on hybridizing variations of rough computing with different concepts in various applications. Similarly, very limited number of rough computing applications are found in ambient intelligence, cultural heritage, and text processing. Besides, rough computing is rarely integrated with structural equation modeling and partial least square while analyzing primary data. Therefore, there is a lot of scope in rough computing research so far data analytics is concerned.

CRediT authorship contribution statement

D.P. Acharjya: Written this paper based on his research interest, rough computing. Rough computing has gone a long way. But, a standard review on rough computing is not available. Keeping it in mind, how the abstraction has been carried out in rough set and its applications are highlighted. **Ajith Abraham:** Reviewed the paper carefully since its drafting stage, The software libraries of rough set theory is highlighted, Thrown light to expand the various applications of rough computing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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